第 10~11 週進度 轉動 I~II 會考題庫, Oct, 2017

對應 R. Wolfson (RW): Essential University Physics 3e (EUP3e)章節: 第 10 章 轉動 (rotational motion) 與第 11 章轉動向量與角動量 (rotational vectors & angular momentum) 會考題庫, Oct, 2017

[1.中] 當一個小朋友站在旋轉木馬的邊緣上, 以下何者是對的?
(a) 如果旋轉木馬具有等角速度, 則小朋友必然有切線加速度;
(b) 即使旋轉木馬具有等角速度,小朋友也仍然**沒有**任何**線**加速度;
(c) 即使旋轉木馬的角加速度呈線性增加, 小朋友仍然**沒有**切線加速度;
(d) 如果旋轉木馬的角加速度呈線性增加, 小朋友就會有切線加速度.

[1.英] A child standing at the rim of a *merry-go-round* (i.e. *roundabout* in British English).
(a) if the merry-go-round is revolving with a constant angular velocity, the child must have tangential acceleration;
(b) even if the merry-go-round is revolving with a constant angular velocity, the child must **NOT** have any **LINEAR** acceleration;
(c) even if the angular velocity increases uniformly, the child still does **NOT** have tangential acceleration;
(d) if the angular velocity increases uniformly, the child must have tangential acceleration.

旋轉木馬遊具 (merry-go-round or roundabout)

Answer: (d).
解說: (a)(b)等角速度 (等速率) 圓周運動必需有且只有向心加速度$a_c = v^2/r$;
(c)(d)有角加速度就會有切線加速度$a_t = dv/dt = r\, d\omega/dt$ (若$v, \omega \neq 0$則同時也有向心加速度$a_c = v^2/r$).

[2.中] 以下敘述何者爲眞?
(a) 若有 2 個力作用在同個物體上且合力是$\vec{\mathbf{0}}$, 則該 2 力的合力矩也必定是$\vec{\mathbf{0}}$;
(b) 即使 2 個力作用在同個物體上且合力是$\vec{\mathbf{0}}$, 則該 2 力的合力矩也**未必**是$\vec{\mathbf{0}}$;

(c) 我們可以把逆時鐘方向的力矩（力矩向量指出紙面）加在順時鐘轉的物體上，且物體的角速率會先變快;

(d) 我們**無法**把逆時鐘方向的力矩（力矩向量指出紙面）加在順時鐘轉的物體上.

[2.英] Which of the following descriptions is correct?

(a) If two forces act on the same object but the net force is $\vec{\mathbf{0}}$, the net torque must be $\vec{\mathbf{0}}$;

(b) Even if two forces act on the same object but the net force is $\vec{\mathbf{0}}$, the net torque may possibly **NOT** $\vec{\mathbf{0}}$;

(c) We can apply a counterclockwise torque to an object that's rotating clockwise, and the object will rotate faster in the beginning;

(d) We can **NEVER** apply any counterclockwise torque to an object that's rotating clockwise.

Answer: (b).

解說: (a)(b)有任 2 力向量和是$\vec{\mathbf{0}}$，若作用在同個點上，則合力矩必定是$\vec{\mathbf{0}}$，若作用在不同點上，則合力矩必定非$\vec{\mathbf{0}}$; (c) 會先變慢，停止後再逆轉變快，例如等角加速度時$\omega = \omega_0 + \alpha t = \omega_0 - |\alpha| t$; (d) 爲僞.

[3.中] 對於在水平地板上質量相同，整體的球半徑相同，滾動且無打滑的實心球與空心球等 2 個物體，下列敘述何者爲眞?

(a) 若 2 個物體具有相同的總動能，則 2 者滾動一樣快;

(b) 若 2 個物體具有相同的總動能，則空心球滾動較快;

(c) 若 2 個物體具有相同的速率，則 2 者所具有的總動能相同;

(d) 若 2 個物體具有相同的速率，則空心球所具有的總動能較大.

[3.英] Consider a solid sphere and a hollow sphere of the same mass and radius are rolling along level ground. Which of the following descriptions is correct?

(a) If they have the same total kinetic energy, they move at the same speed;

(b) If they have the same total kinetic energy, the hollow sphere is moving faster;

(c) If they are moving at the same speed, they have the same total kinetic energy;

(d) If they are moving at the same speed, the hollow sphere has more kinetic energy.

Answer: (d).

解說：質量M相同且以過球心的軸當轉軸時，2 球有類似的轉動慣量形式$I_{\rm cm} = \kappa MR^2$，即$\kappa = I_{\rm cm}/(MR^2)$，其中空心球$I_{\rm cm,hol} = \kappa_{\rm hol} MR^2 = 2MR^2/3$比實心球的轉動慣量$I_{\rm cm,sol} = \kappa_{\rm sol} MR^2 = 2MR^2/5$大，空心球的$\kappa$值$\kappa_{\rm hol} = 2/3$較實心球的$\kappa_{\rm sol} = 2/5$大（附帶一提，空心球的殼層上的密度也較實心球大）. (a)(b) 無

打滑的滾動$v_{\rm cm} = R\omega$，總動能是

$$K_{\rm tot} = K_{\rm cm} + K_{\rm rot} = \frac{1}{2}Mv_{\rm cm}^2 + \frac{1}{2}I_{\rm cm}\omega^2 = \frac{1}{2}Mv_{\rm cm}^2 + \frac{1}{2}I_{\rm cm}\left(\frac{v_{\rm cm}^2}{R}\right)^2$$

$$= \frac{1}{2}\left(M + \frac{I_{\rm cm}}{R^2}\right)v_{\rm cm}^2 = \frac{1}{2}\left(M + \frac{I_{\rm cm}}{R^2}\right)R^2\omega^2 = \frac{1}{2}(MR^2 + I_{\rm cm})\omega^2,$$

在$K_{\rm tot}$相同時，$I_{\rm cm}$較大者（空心球）速率與角速率等 2 者皆較小，因此空心球的速率比較小；(c)(d) 無滑動的滾動，轉動動能$K_{\rm rot} = I_{\rm cm}\omega^2/2$與轉動慣量$I_{\rm cm}$成正比，平移動能$K_{\rm cm} = Mv_{\rm cm}^2/2$在(c)(d)的條件下相同，因此空心球的總動能比較大.

[4.中] 把空轉中的圓鋸的電關掉之後，有正常安裝圓鋸片在轉軸上的情形，會比轉軸上沒有裝圓鋸片的情形轉更久纔停止. 若與未安裝圓鋸片的情形比較，安裝圓鋸片之後，以下描述何者正確?

(a) 有裝圓鋸片時，整體的轉動慣量較小；

(b) 若兩系統以相同的角速度旋轉，則有裝圓鋸片時的轉動動能比較小；

(c) 若分別施加相同的力矩在兩系統上，則有裝圓鋸片時的角速度變化較慢（即角加速度向量的大小較小，不討論方向)；

(d) 若以相同的角速度旋轉，則有裝圓鋸片時的角動量大小比較小.

[4.英] A circular saw takes a long time to stop rotating after the power is turned off. Without the saw blade mounted, the motor stops much more quickly. In comparison with the system without the saw blade mounted, which of the following descriptions is correct about the system with saw blade mounted?

(a) The system with the saw blade mounted has smaller rotational inertia;

(b) If the two systems rotates at the same angular speed, the system with the saw blade mounted has smaller rotational kinetic energy;

(c) If we apply the same torque on two systems, the rotational speed of the system with the saw blade mounted varied more slowly (i.e., the amount of the angular acceleration is smaller, without mentioning its direction);

(d) If the two systems rotates at the same angular speed, the system with the saw blade mounted has smaller angular speed.

[會考時可將照片省略]

Answer: (c).

解說: (a) 轉動慣量I是疊加的, 且貢獻恆不爲負, 追加物品在系統上不可能使轉動慣量變小, 且依經驗, 鋸片的轉動慣量顯然比轉軸還要大許多 (也因爲鋸片的質量分佈比轉軸要遠許多);
(b) 角速度ω相同, 則角動能$K_{\mathrm{rot}} = I\omega^2/2$與轉動慣量$I$成正比;
(c) 角加速度向量的大小$\alpha = \tau/I$與轉動慣量I成反比;
(d) 角速度ω相同, 則角動量的大小$L = I\omega$與轉動慣量I成正比.

[5.中] 考慮地球自轉與汽車行進時的角速度向量, 以下描述何者正確?
(a) 地球相對於地軸的自轉角速度向量方向向**北**;
(b) 地球相對於地軸的自轉角速度向量方向向**南**;
(c) 當汽車**前進**時, 車輪相對於輪軸的角速度向量向前;
(d) 當汽車**後退**時, 車輪相對於輪軸的角速度向量向前.

[5.英] Consider the Earth's and a vehicle's angular velocity vectors. Which of the flowing descriptions is correct?
(a) the Earth's angular velocity vector point **NORTH**;
(b) the Earth's angular velocity vector point **SOUTH**;
(c) when a vehicle is moving **FORWARD**, the wheel's angular velocity vector point forward;
(d) when a vehicle is moving **BACKWARD**, the wheel's angular velocity vector point forward.

Answer: (a).

解說: (a)(b) 地球自轉時由西向東轉, 依右手系的規定, 角速度方向向北; (c)(d) 汽車前進時車輪的角速度向左, 後退時車輪的角速度向右.

[6.中] 當你站著且將右手向右水平伸出, 則地球重力相對於肩膀所造成的力矩方向是向何方? (a) 前; (b) 後; (c) 上; (d) 下.

[6.英] You stand with your right arm extended horizontally to the right. What's the direction of the gravitational torque about your shoulder?
(a) forward; (b) backward; (c) upward; (d) downward.

Answer: (a).

解說: 力臂$\vec{\mathbf{r}}$向右, 重力$\vec{\mathbf{F}}_{\mathrm{g}}$向下, 因此重力所施力矩$\vec{\boldsymbol{\tau}}_{\mathrm{g}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}_{\mathrm{g}}$向前.

[7.中] 若有 2 個大小固定但是可以調節方向的向量，以下何者描述正確?
(a) 當 2 個向量互相**平行**時，外積是$\vec{\mathbf{0}}$;
(b) 當 2 個向量互相**垂直**時，外積是$\vec{\mathbf{0}}$;
(c) 當 2 個向量互相**平行且同向**時，外積的大小最大;
(d) 當 2 個向量互相**平行且反向**時，外積的大小最大.

[7.英] Without knowing any magnitudes, which of the following descriptions is true?
(a) when the two vectors parallel to each other, their outer product is $\vec{\mathbf{0}}$;
(b) when the two vectors perpendicular to each other, their outer product is $\vec{\mathbf{0}}$;
(c) when the two vectors are parallel to each other and in the **SAME** direction, their outer product has its maximum;
(d) when the two vectors are parallel to each other and in the **OPPOSITE** direction, their outer product has its maximum.

Answer: (a).

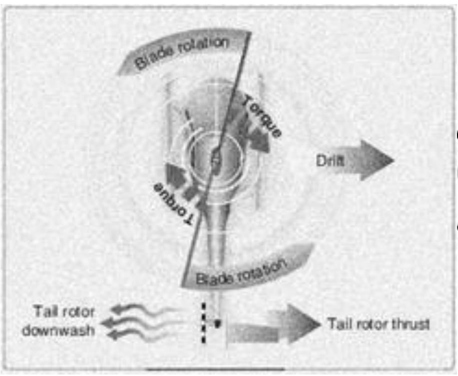
解說: 2 個向量互相平行時，向量間張角的正弦值$\sin\theta = 0$，外積是$\vec{\mathbf{0}}$，外積的大小最小; 2 個向量互相垂直時，向量間張角的正弦值$\sin\theta = 1$，外積的大小最大.

[8.中] 考慮直升機的 2 個旋翼 (螺旋槳)，第 2 個旋翼是爲了平衡第 1 個旋翼所產生的 (a) 升力; (b) 重力; (c) 力矩; (d) 轉動動能.

[8.英] The reason why do helicopters have two rotors, is to use the 2nd rotor to balance which effect generated by the 1st rotor?
(a) lift force; (b) gravitational force; (c) torque; (d) rotational kinetic energy.

Answer: (c).

解說: 第 1 個旋翼提供升力的同時，會施加力矩在機身上，由於直升機沒有一般飛機那樣大的固定機翼，機身無法單靠通過機身的空氣提供足夠的反方向力矩，必需利用第 2 個旋翼所產生的力矩維持機身穩定. 旋翼的大小則因設計而不同.



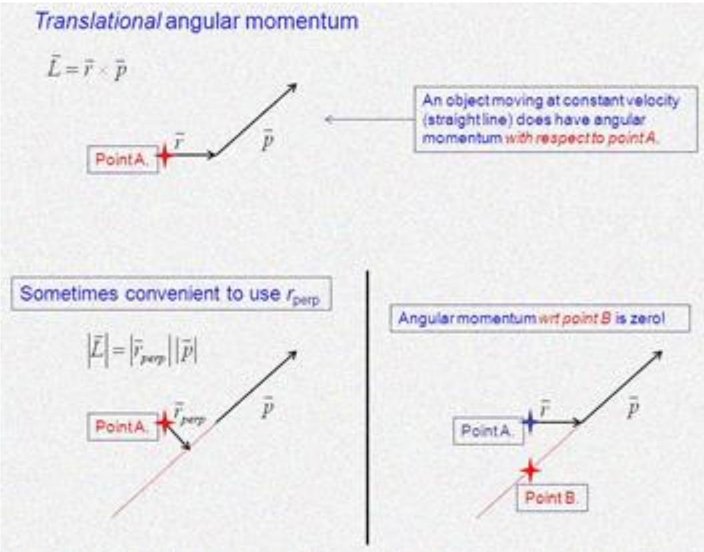
[解說用圖無需加在題目上].

[9.中] 某個沒有自轉的物體以等速率沿著直線移動，以下的敘述何者正確?
(a) 對於該直線**上**的點，該物體仍然具有非零的角動量;
(b) 對於該直線**上**的點，該物體的角動量**不**是定值;
(c) 對於該直線**外**的點，該物體仍然具有非零的角動量;
(d) 對於該直線**外**的點，該物體的角動量**不**是定值.

[9.英] If a particle without any rotation is moving at a constant speed in a straight line, which of the following descriptions is correct?
(a) it has nonzero momentum about a point on the line;
(b) its angular momentum is **NOT** a constant about a point on the line;
(c) it has nonzero momentum about a point **NOT** on the line;
(d) its angular momentum about a point **NOT** on the line is **NOT** a constant.

Answer: (c).

解說：等速度運動的質點，線動量向量$\vec{\mathbf{p}}$是定值. (a)(b) 對於軌跡直線上的點，質點的相對位置恆與線動量向量平行或反平行，因此角動量向量恆是定值$\vec{\mathbf{0}}$; (c)(d) 對於軌跡直線外的點，相對位置向量$\vec{\mathbf{r}}$在垂線上的投影長$r\sin\theta$是定值，因此$\vec{\mathbf{r}}\times\vec{\mathbf{p}}$的大小$rp\sin\theta$是定值，觀察圖形可知$\vec{\mathbf{r}}\times\vec{\mathbf{p}}$的方向亦固定 (如下圖例中是在出紙面方向)，因此$\vec{\mathbf{r}}\times\vec{\mathbf{p}}$是定值.



[解說用圖無需加在題目上].

[10.中] 某 2 個隨意指定大小的向量，其內積等於其外積的大小. 則 2 個向量間的夾角應該是(a) 0°; (b) 45°; (c) 90°; (d) 180°.

[10.英] The dot product of two vectors is equal to the magnitude of their cross product. What can you conclude about the angle between them? (a) 0°; (b) 45°; (c) 90°; (d) 180°.

Answer: (b).

解說: 對於任意大小的 2 個向量$\vec{\mathbf{a}}$與$\vec{\mathbf{b}}$, $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = ab\cos\theta$, $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = ab\sin\theta$, 兩者相等則$\cos\theta = \sin\theta$, 夾角$\theta = 45°, 135°, \ldots$

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[11.中] 地球自轉一周需時 23.93 小時 (即 1 個恆星日). 所以自轉的角速率是多大?

(a) 1.16×10^{-5} rad/s; (b) 0.042 rad/s; (c) 1.39×10^{-5} rad/s;
(d) 7.29×10^{-5} rad/s.

[11.英] Earth makes one rotation in 23.93 hours (i.e., a sidereal day). What's its angular speed?

(a) 1.16×10^{-5} rad/s; (b) 0.042 rad/s; (c) 1.39×10^{-5} rad/s;
(d) 7.29×10^{-5} rad/s.

Answer: (d).

解說: 1 個恆星日是 23.93 hr = 86148 s, 角速率
$\omega = (2\pi\ \text{rad})/(86148\ \text{s}) = 7.293 \times 10^{-5}$ rad/s.

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[12.中] 要在 9.0 s 之內把以 6.0 rev/s 旋轉的輪子停下, 所需的平均角加速度是以下何者? (a) -0.67 rad/s^2; (b) -2.1 rad/s^2; (c) -4.2 rad/s^2; (d) -67 rad/s^2.

[12.英] To stop a wheel rotating at 6.0 rev/s in 9.0 s requires average angular acceleration

(a) -0.67 rad/s^2; (b) -2.1 rad/s^2; (c) -4.2 rad/s^2; (d) -67 rad/s^2.

Answer: (c).

解說: 初角速率$\omega_\text{i} = 6.0\ \text{rev/s} = 6.0\ (2\pi\ \text{rad})/\text{s} = 12.0\pi\ \text{rad/s}$, 角速率變化

$$\Delta\omega = 0 - \omega_\text{i} = \alpha\,\Delta t, \alpha = -\frac{\omega_\text{i}}{\Delta t} = -\frac{(12.0\pi\ \text{rad/s})}{(9.0\ \text{s})} = -4.189\ \text{rad/s}^2.$$

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[13.中] 轆轤 (拉胚用的轉盤) 開始時以角速率 2.4 rad/s 旋轉, 在時距 2.0 s 之內以等角加速至 3.6 rad/s. 在角加速這段期間, 轆轤轉了多少角度?

(a) 6.0 rad; (b) 12.0 rad; (c) 0.95 rad; (d) 4.8 rad.

[13.英] A potter's wheel starting with angular speed 2.4 rad/s accelerates in 2.0 s to 3.6 rad/s, with constant angular acceleration. During this time the wheel turns through an angle of (a) 6.0 rad; (b) 12.0 rad; (c) 0.95 rad; (d) 4.8 rad.

Answer: (a).

解說: 角速度大小的變化$\Delta\omega = \omega_f - \omega_i = \alpha\,\Delta t$, 角加速度

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(3.6 - 2.4)\ \text{rad/s}}{(2.0\ \text{s})} = 0.6\ \text{rad/s}^2,$$

$$\text{角位移}\ \Delta\theta = \omega_i\,\Delta t + \frac{1}{2}\alpha\,(\Delta t)^2 = (2.4\ \text{rad/s})(2.0\ \text{s}) + \frac{1}{2}(0.6\ \text{rad/s}^2)(2.0\ \text{s})^2$$

$= 4.8\ \text{rad} + 1.2\ \text{rad} = 6.0\ \text{rad}$.

[14.中] 一片半徑 6.0 cm 的 CD 由靜止等角加速至 40 rad/s 花了 2.5 s, 則 CD 邊緣的切線加速度大小是多少?

(a) 0.49 m/s^2; (b) 0.96 m/s^2; (c) 0.22 m/s^2; (d) 6.0 m/s^2.

[14.英] A compact disk with radius 6.0 cm takes 2.5 s to accelerate from rest to 40 rad/s. What's the tangential acceleration of a point on the CD's edge?

(a) 0.49 m/s^2; (b) 0.96 m/s^2; (c) 0.22 m/s^2; (d) 6.0 m/s^2.

Answer: (b).

解說: 角速度大小的變化$\Delta\omega = \omega_f - 0 = \alpha\,\Delta t$, 角加速度

$\alpha = \frac{\omega_f}{\Delta t} = \frac{(40\ \text{rad/s})}{(2.5\ \text{s})} = 16\ \text{rad/s}^2$, 切線加速度大小

$a_t = r\alpha = (6.0 \times 10^{-2}\ \text{m})(16\ \text{rad/s}^2) = 0.96\ \text{m/s}^2$.

[15.中] 有一輪以等角加速度旋轉, 則以下何者的大小是定值?

(a) 角速度; (b) 切線速率; (c) 切線加速度; (d) 向心加速度.

[15.英] A wheel rotates with constant angular acceleration. Which of the following amount is constant?

(a) angular velocity; (b) tangential speed; (c) tangential acceleration;

(d) centripetal acceleration.

Answer: (c).

解說: (a)(b) 角速度$\omega = \omega_0 + \alpha t$與切線速率$v = r\omega = r\omega_0 + r\alpha t = v_0 + a_t t$依線性增加; (c) 切線加速度$a_t = r\alpha$與角加速度$\alpha$成正比, 保持定值; (d) 向心加速度$a_c = v^2/r = (v_0 + a_t t)^2/r = r\omega^2 = (r)(\omega_0 + \alpha t)^2$與切線速率的平方或角速度的平方成正比, 因此依平方形式增加.

[16.中] 一碟具有半徑 1.5 m 且轉動慣量是 34 $\text{kg}\cdot\text{m}^2$, 受到 160 N 的力作用在邊緣上. 如果碟由靜止開始旋轉, 在 2.0 s 過後其角速率是
(a) 1.22 rad/s; (b) 6.27 rad/s; (c) 7.06 rad/s; (d) 14.1 rad/s; (e) 25.5 rad/s.
[16.英] A disk with radius 1.5 m and rotational inertia 34 $\text{kg}\cdot\text{m}^2$ has a 160-N force applied tangentially to its rim. If the disk starts from rest, the angular speed at the end of 2.0 s is
(a) 1.22 rad/s; (b) 6.27 rad/s; (c) 7.06 rad/s; (d) 14.1 rad/s; (e) 25.5 rad/s.
Answer: (d).
解說: 依轉動的 Newton 第二運動定律, 力矩的大小$\tau = I\alpha$, 角加速度是
$\alpha = \tau/I = rF/I = (1.5\text{ m})(160\text{ N})/(34\text{ kg}\cdot\text{m}^2) = 7.059\text{ rad/s}^2$,

$$\omega_\text{f} = 0 + \alpha t = (7.059\text{ rad/s}^2)(2.0\text{ s}) = 14.12\text{ rad/s}$$

[17.中] 有個 46 克的球具有 2.13 cm 的半徑. 設球是均質的, 則球的轉動慣量是多少?
(a) 8.3×10^{-6} $\text{kg}\cdot\text{m}^2$; (b) 2.1×10^{-5} $\text{kg}\cdot\text{m}^2$; (c) 1.3×10^{-5} $\text{kg}\cdot\text{m}^2$;
(d) 4.3×10^{-6} $\text{kg}\cdot\text{m}^2$.
[17.英] A 46-g golf ball has radius 2.13 cm. Assuming uniform density, what's the ball's rotational inertia?
(a) 8.3×10^{-6} $\text{kg}\cdot\text{m}^2$; (b) 2.1×10^{-5} $\text{kg}\cdot\text{m}^2$; (c) 1.3×10^{-5} $\text{kg}\cdot\text{m}^2$;
(d) 4.3×10^{-6} $\text{kg}\cdot\text{m}^2$.
Answer: (a).
解說: 均質實心球對於過球心的軸轉動慣量是
$I_\text{cm} = 2MR^2/5 = (2)(46\times10^{-3}\text{ kg})(2.13\times10^{-2}\text{ m})^2/5 = 8.348\times10^{-6}\text{ kg}\cdot\text{m}^2$.

[18.中] 有個球具有8.35×10^{-6} $\text{kg}\cdot\text{m}^2$的轉動慣量, 若該球以 80 Hz 自轉, 則自轉的動能是多少? (a) 0.5 J; (b) 1.0 J; (c) 2.0 J; (d) 4.0 J.
[18.英] A golf ball has rotational inertia 8.35×10^{-6} $\text{kg}\cdot\text{m}^2$, what's the rotational kinetic energy when it's rotating at 80 Hz?
(a) 0.5 J; (b) 1.0 J; (c) 2.0 J; (d) 4.0 J.
Answer: (b).
解說: $\omega = 2\pi f = (2\pi)(80\text{ Hz}) = 160\pi\text{ rad/s}$,

$$K_{\rm rot} = \frac{I\omega^2}{2} = \frac{(8.35\times10^{-6}\ \rm kg\cdot m^2)(160\pi\ rad/s)^2}{2} = 1.055\ \rm J.$$

[19.中] 自行車具有直徑是 69 cm 的輪子，且以 40 km/h 的速率移動，其輪子滾動時與地面之間沒有打滑，則輪子的角速率是

(a) 4 rad/s; (b) 9 rad/s; (c) 16 rad/s; (d) 32 rad/s.

[19.英] A bicycle with wheels 69 cm in diameter is moving at 40 km/h. If the wheels roll without slipping, their angular speed is

(a) 4 rad/s; (b) 9 rad/s; (c) 16 rad/s; (d) 32 rad/s.

Answer: (d).

解說：無打滑的滾動其平移速率是$v_{\rm cm} = \omega R$，

$$\omega = \frac{v_{\rm cm}}{R} = \frac{40\ \rm km/h}{(69\ cm)/2} = \frac{(40)(10^3\ \rm m)/(3600\ s)}{(69\times10^{-2}\ \rm m)/2} = 32.21\ \rm rad/s.$$

[20.中] 實心圓柱具有質量 0.55 kg 與半徑 3.5 cm，以質心速率 0.75 m/s 進行無打滑的滾動，則其總共的動能是多少? (a) 0.007 J; (b) 0.16 J; (c) 0.23 J; (d) 0.30 J.

[20.英] A solid cylinder has mass 0.55 kg and radius 3.5 cm. It's rolling with center-of-mass speed 0.75 m/s. What's its total kinetic energy?

(a) 0.007 J; (b) 0.16 J; (c) 0.23 J; (d) 0.30 J.

Answer: (c).

解說：實心圓柱沿縱向對稱軸轉動的轉動慣量是$I_{\rm cm} = MR^2/2$，無打滑的滾動其平移速率是$v_{\rm cm} = \omega R$，$\omega = v_{\rm cm}/R$，總動能是

$$K = \frac{1}{2}Mv_{\rm cm}^2 + \frac{1}{2}I_{\rm cm}\omega^2 = \frac{1}{2}Mv_{\rm cm}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{\rm cm}^2}{R}\right)^2 = \frac{1}{2}Mv_{\rm cm}^2 + \frac{1}{4}Mv_{\rm cm}^2$$

$$= \frac{3}{4}Mv_{\rm cm}^2 = \frac{3}{4}(0.55\ \rm kg)(0.75\ m/s)^2 = 0.2320\ J.$$

[21.中] 在汽車的傳動系統的離合器 (clutch) 上，具有轉動慣量 26.0 $\rm kg\cdot m^2$的飛輪以 310 rad/s 的角速率轉動. 若將離合器咬合，將離合板--具有飛輪一半轉動慣量的碟--壓在飛輪上，使得 2 者一體轉動. 假設飛輪-離合板系統不受外部的力矩影響，則 2 者咬合之後角速率是多少?

(a) 155 rad/s; (b) 206 rad/s; (c) 267 rad/s; (d) 310 rad/ s.

[21.英] (AR&RW:ECP1e 英文首版:ch 8:MCP:27.) In a car's drive train, a flywheel with rotational inertia 26.0 $\mathrm{kg \cdot m^2}$ rotates at 310 rad/s. The clutch engages, pressing the clutch plate—a disk with rotational inertia half that of the flywheel—against the flywheel, so the two rotate as one. Assuming both are otherwise free from torque, what's the rotational speed of the combined system?
(a) 155 rad/s; (b) 206 rad/s; (c) 267 rad/s; (d) 310 rad/ s.

Answer: (b).

解說：系統不受外力矩作用，則整體的角動量守恆，咬合前後的角動量相等，因爲咬合前後的轉動慣量間關係是$I_\mathrm{i} = I_\mathrm{fw} = 26.0\ \mathrm{kg \cdot m^2}$，咬合後的轉動慣量是

$$I_\mathrm{f} = I_\mathrm{fw} + I_\mathrm{cp} = I_\mathrm{fw} + \frac{1}{2}I_\mathrm{fw} = \frac{3}{2}I_\mathrm{fw},\text{不受外力矩時角動量守恆}I_\mathrm{i}\omega_\mathrm{i} = I_\mathrm{f}\omega_\mathrm{f} = \frac{3}{2}I_\mathrm{f}\omega_\mathrm{f}$$

$$\omega_\mathrm{f} = \frac{2}{3}\omega_\mathrm{i} = \frac{2}{3}(310\ \mathrm{rad/s}) = 206.7\ \mathrm{rad/s}.$$

[22.中] 旋轉中的輪可能處於以下何種情形?
(a) 兼有向心加速度與切線加速度; (b) 沒有向心加速度也沒有切線加速度;
(c) 只有角加速度卻沒有向心加速度; (d) 沒有角加速度也沒有向心加速度.

[22.英] (AR&RW:ECP1e 英文首版:ch 8:MCP:28.) A rotating wheel may have
(a) both centripetal and tangential acceleration;
(b) neither centripetal nor tangential acceleration;
(c) angular but not centripetal acceleration;
(d) neither angular nor centripetal acceleration.

Answer: (a).

解說：只要有旋轉，角速度$\vec{\boldsymbol{\omega}}$必不是$\vec{\mathbf{0}}$，向心加速度$\vec{\mathbf{a}}_\mathrm{c}$也必不是$\vec{\mathbf{0}}$（其大小是$a_\mathrm{c} = v^2/r = r\omega^2$），所以(b)(c)(d)不可能；如果維持在同個轉軸上旋轉，但角速率ω（俗稱轉速）在變化，則會有角加速度$\vec{\mathbf{a}}_\mathrm{c}$以及切線加速度$\vec{\mathbf{a}}_\mathrm{t}$（其大小是$a_t = \mathrm{d}v/\mathrm{d}t = r\ \mathrm{d}\omega/\mathrm{d}t$）(如果討論更複雜的情形，則轉軸的方向也會改變).

[23.中] 有個巨輪具有轉動慣量 25 $\mathrm{kg \cdot m^2}$與半徑 0.75 m，最初靜止，後以 35 N 的力在邊緣上作用 5.0 s，求最終的角速率.
(a) 086 rad/s; (b) 1.7 rad/s; (c) 5.3 rad/s; (d) 10.6 rad/s.

[23.] (AR&RW:ECP1e 英文首版:ch 8:MCP:25.) A heavy machine wheel has a rotational inertia 25 $\mathrm{kg \cdot m^2}$ and radius 0.75 m. It's initially at rest, and a tangential force of 35 N is applied at its edge for 5.0 s. What's the resulting angular speed? (a) 086 rad/s; (b) 1.7 rad/s; (c) 5.3 rad/s; (d) 10.6 rad/s.

Answer: (c).

解說: 力矩$\tau = RF = I\alpha$, 角加速度$\alpha = RF/I$, 末角速率
$\omega_{\mathrm{f}} = 0 + \alpha\,\Delta t = R\,F\,\Delta t/I = (0.75\ \mathrm{m})(35\ \mathrm{N})(5.0\ \mathrm{s})/(25\ \mathrm{kg}\cdot\mathrm{m}^2) = 5.25\ \mathrm{rad/s}$.

[24.中] 月球具有質量7.35×10^{22} kg, 每 27.3 天公轉 1 周, 軌道半徑是3.84×10^{8} m, 試計算月球公轉的角動量.

(a) 7.33×10^{25} kg·m²/s; (b) 2.81×10^{34} kg·m²/s;
(c) 7.33×10^{41} kg·m²/s; (d) 7.33×10^{51} kg·m²/s.

[24.英] The Moon (mass 7.35×10^{22} kg) takes 27.3 days to complete an essentially circular orbit of radius 3.84×10^{8} m. Estimate the magnitude of the Moon's orbital angular momentum.

(a) 7.33×10^{25} kg·m²/s; (b) 2.81×10^{34} kg·m²/s;
(c) 7.33×10^{41} kg·m²/s; (d) 7.33×10^{51} kg·m²/s.

Answer: (b).

解說: 月球軌道半徑遠大於其本身的半徑, 可以質點近似計算. 若以不動的圓形軌道近似計算, 相對於軌道中心點, 其角動量是

$$L = rMv = rMr\omega = r^2M\omega = r^2M2\pi/T$$
$$= (3.84\times10^{8}\ \mathrm{m})^2(7.35\times10^{22}\ \mathrm{kg})(2\pi)/(27.3\times86400\ \mathrm{s})$$
$$= 2.887\times10^{34}\ \mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}$$

[25.中] 有一輪, 具有鉛直轉軸, 且由上方觀看時是逆時鐘旋轉, 則輪的角動量向量是向 (a) 鉛直向上; (b) 鉛直向下; (c) 沿輪的切線方向, 且與旋轉方向相同; (d) 沿輪的切線方向, 且與旋轉方向**相反**.

[25.英] (AR&RW:ECP1e 英文首版:ch 8:MCP:29.) A wheel has its rotation axis vertical, and turns counterclockwise as viewed from above. The direction of the wheel's angular momentum is (a) straight up; (b) straight down; (c) tangent to the wheel, in the direction of rotation; (d) tangent to the wheel, **opposite to** the direction of rotation.

Answer: (a).

解說: 角速度向量$\vec{\boldsymbol{\omega}}$與角動量向量$\vec{\mathbf{L}}$的方向皆依右手系的規定.