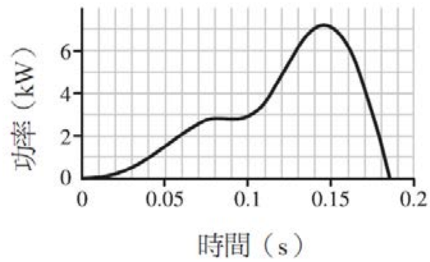
1. 你用 200 N 的力量沿著 30°角的傾斜面以等速率推一個裝書的箱子,如果摩擦係數為 0.18,則(1)當箱子垂直上升 1 m 時你做了多少功? (A) 200 J, (B) 400 J, (C) 600 J, (D) 800 J; (2)箱子的質量為多少? (A) 15.5 kg, (B) 31.0 kg, (C) 46.5 kg, (D) 62.0 kg.
2. 如果作用力$\vec{F} = (67\hat{\imath} + 23\hat{\jmath} + 55\hat{k})$N 施加於一個物體使它沿著直線從位置$\vec{r_1} = (16\hat{\imath} + 31\hat{\jmath})$m 移動到$\vec{r_2} = (21\hat{\imath} + 10\hat{\jmath} + 14\hat{k})$m,則所做的功為何? (A) 335 J, (B) 483 J, (C) 622 J, (D) 770 J.
3. 一部 0.5 馬力(1 hp = 746 W)之井底抽水機要將水輸送到離井底水面 80 m 高的水槽中,求水的輸送率為何?(將答案以 kg/s 的單位表示) (A) 373 kg/s, (B) 38.1 kg/s, (C) 0.47 kg/s, (D) 0.94 kg/s.
4. 一輛 1400 kg 的車子以 60 km/h 的固定速率沿著山路往上移動,汽車所受到的空氣阻力為 550 N,如果引擎動力傳送到帶動輪的功率為 38kW,求山路的坡度為何? (A) 7.2°, (B) 14.4°, (C) 21.6°, (D) 28.8°
5. 被打擊出去的棒球其能量來自於球棒所做的功,最強有力的打擊者可在短暫的接觸時間內提供大約 10 馬力(1 hp = 746 W)的打擊功率,使棒球飛出去的速率超過每小時 100 英哩,下圖顯示某個打擊功率相對於時間的變化情形。



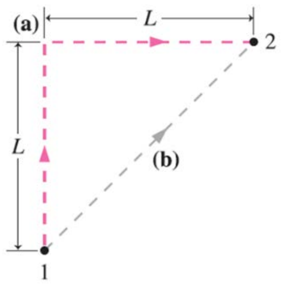
(1) 在曲線的峰值時,下面的選項何者最大?
 (A) 球之動能
 (B) 球之速率
 (C) 球棒對球所做的功
 (D) 球棒對球所提供的能量效率

(2) 球的最大速率大約在時間為多少秒時?
 (A) 85 ms
 (B) 145 ms
 (C) 185 ms
 (D) 當力量最大時

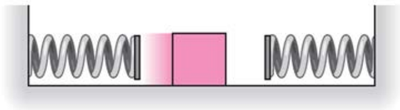
(3) 棒球被擊出後其動能加了大約多少?
 (A) 550J
 (B) 1.3 kJ
 (C) 7.0 kJ
 (D) 無法計算,因為不知道投手將棒球投出來的速率

(4) 施 加 於 棒 球 的 作 用 力 在 何 時 最 大 ?
(A) 在 185ms
(B) 在圖中的峰值前
(C) 在圖中的峰值時
(D) 在圖中的峰值後與 185ms 前

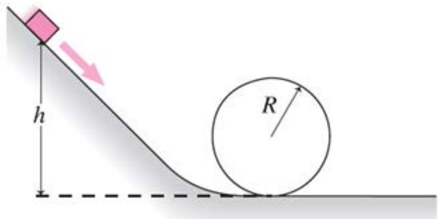
6. 在龍捲風過後,一支 0.50 g 吸管被插入一棵樹中,深度為 4.5 cm,測量顯示樹枝對吸管的阻擋力為 70 N,求吸管插入樹枝前的速率? (A) 27.5 m/s, (B) 55 m/s, (C) 82.5 m/s, (D) 110 m/s.
7. 某工人在水平地面上推動一個質量 180 kg 的行李箱時,碰到一段長 10 m 粗糙地板,此地板的粗糙程度隨著位置的增加而變大，其動摩擦係數為$\mu_k = \mu_0 + ax^2$,其中 $\mu_0 = 0.17, a = 0.0062m^{-2}$, x代表從粗糙地板起算點到所考慮之位置的距離，如果工人以等速度推動行李箱越過此粗糙區域，則他所做的功為多少？ (A) 6.6 kJ, (B) 9.9 kJ, (C) 13.2 kJ, (D) 19.8 kJ.
8. 某拖船施加作用力$\vec{F} = (1.2\hat{\imath} + 2.3\hat{\jmath})$MN， 沿一條直線的航道拉一艘遊艇行進一個位移 $\Delta\vec{r} = (380\hat{\imath} + 460\hat{\jmath})$m.求(1) 拖船所做的功? (A) 755 MJ, (B) 1510 MJ, (C) 2265 MJ, (D) 3120 MJ; (2)作用力與位移的夾角? (A) 12°, (B) 24°, (C) 36°, (D) 48°
9. 在中午時刻,太陽能輻射到地球表面的功率大約為 1 kW/m^2,對一個面積為 15 m^2 之完美的太陽能板而言,它要花多久的時間才能收集 40 kW・h 的能量?(這大約是 1 加侖汽油所擁有的能量) (A) 1.6h, (B) 2.0h, (C) 2.4h, (D) 2.7h
10. 高空彈跳所使用之彈力繩索的長度通常是 11 m,彈性常數則為 k = 250 N/m,如果彈跳者抵達下降之最底端時,繩索長度變為原來的兩倍.求(1)作用於繩索的功？(A) 5kJ, (B) 10 kJ, (C) 15 kJ, (D) 20 kJ. (2)使繩索伸長最後 1m 所做的功,比起最初 1 m 所做的功,比值大小為何? (A) 11, (B) 21, (C) 31, (D) 41.
11. 某質量為 m 的物體,分別沿著兩條路線從點 1 移動到點 2,如下圖所示,(1)假設本圖位於水平面上,試分別計算摩擦力對此物體所做的功,已知摩擦係數在平面上為固定值 μ。(A) $W_a = -2\mu_k mgL,\ W_b = -\sqrt{2}\mu_k mgL$, (B) $W_a = -mgL$, $W_b = -mgL$, (C) $W_a = -\sqrt{2}\mu_k mgL,\ W_b = -2\mu_k mgL$, (D) $W_a = mgL$, $W_b = mgL$; (2)假設本圖位於垂直面上,當某物體分別沿著圖中兩條路線從點1移動到點 2,試分別計算重力對此物體所做的功。(A) $W_a = -2\mu_k mgL$, $W_b = -\sqrt{2}\mu_k mgL$, (B) $W_a = -mgL,\ W_b = -mgL$, (C) $W_a = -\sqrt{2}\mu_k mgL$, $W_b = -2\mu_k mgL$, (D) $W_a = mgL,\ W_b = mgL$.



12. 一個粒子在光滑的軌道上來回滑動,軌道的高度為水平位置的函數$y = ax^2$,其中 a = 0.95m^{-1},如果粒子的最大速率為 9.2 m/s,則其轉折點的位置為何? (A) ± 0.55 m, (B) ± 1.1 m, (C) ± 1.7 m, (D) ± 2.1 m
13. 用來攀岩的繩索具有相當好的彈性,因此可以做為掉落時的緩衝裝置.有一款繩索之彈力為$F = -kx + bx^2$,其中$k = 223\ N/m$,$b = 4.10\ N/m^2$, x 為伸長量,如果它被拉長 2.62 m,求儲存於繩索之位能.取 x=0 時 U=0. (A) 185 J, (B) 371 J, (C) 556 J, (D) 741 J.
14. 在光滑的水平面上,質量為 100g 的木塊於兩彈簧之間來回滑動,如下圖所示,左邊彈簧的彈性常數為 k = 110 N/m,最大壓縮量為 20 cm,右邊的彈簧則為 k = 240 N/m,求(1)右邊彈簧之最大壓縮量? (A) -7.0 cm, (B) -10.5 cm, (C) -13.5 cm, (D) -16.0 cm; (2)木塊在兩彈簧之間的速率? (A) ± 6.63 m/s, (B) ± 8.63 m/s, (C) ± 10.63 m/s, (D) ± 12.63 m/s

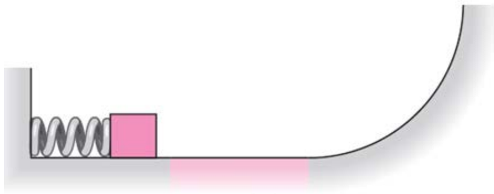


15. 一個木塊沿著斜坡滑下,再接到翻筋斗迴圈滑行,如下圖所示,求木塊從靜止開始滑行而可以繞行一圈之最低高度 h. (A) 3R/2, (B) 2R, (C) 5R/2, (D) 3R.

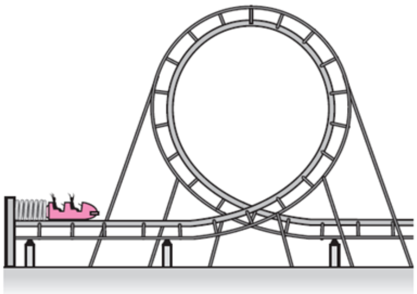


16. 質量為 m 之物體位於彈簧頂端正上方 h 處,彈簧直立於地面其彈性常數為當物體從靜止開始掉落到彈簧上時,找出彈簧的最大壓縮量為:(A) $(2mg/k)(1+\sqrt{1+kh/mg})$, (B) $(2\,mg/k)(1+\sqrt{1+2kh/mg})$, (C) $(mg/k)(1+\sqrt{1+kh/mg})$, (D) $(mg/k)\left(1+\sqrt{1+2kh/mg}\right)$.
17. 某滑雪者沿著光滑的 34°角斜坡滑下山,在垂直下降 24 m 之後,山坡有一短暫距離變得平坦,接著轉為 16°角之斜坡並垂直下降 38 m 之後又變為平坦,求(1)滑雪者在兩處平坦位置的速率? (A) 16.4 m/s, 21.7 m/s, (B) 21.7 m/s, 34.9 m/s, (C) 20 m/s, 30 m/s, (D) 30 m/s, 40 m/s; (2)在斜坡部分的動摩擦係數為 0.11,而水平的部分仍然維持光滑的狀態,則滑雪者在兩處平坦位置的速率? (A) 16.4 m/s, 21.7 m/s, (B) 21.7 m/s, 34.9 m/s, (C) 20 m/s, 30 m/s, (D) 30 m/s, 40 m/s
18. 質量為 182 g 的木塊由彈性常數為$k = 200\ N/m$ 壓縮量為 15 cm 的彈簧發射出去,彈簧位於無摩擦的水平面,但緊鄰彈簧平衡位置為粗糙平面,其摩擦係數為 μ = 0.27,此摩擦平面有 85 cm 長,接著為一向上之光滑的彎曲弧線如下圖所示,木塊被發射後會沿著彎曲弧線上升然後停止,接著滑下來並回到彈簧然後再被發射出去。試求木塊最後停止不動的位置?(從粗糙平面的左邊起算)。(A)

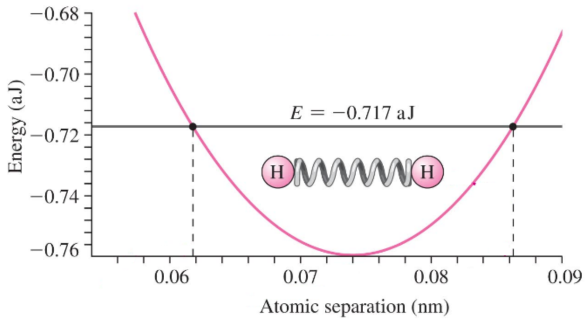
22.9 cm, (C) 34.4 cm, (D) 45.8 cm, (D) 56.4 cm.



19. 質量為 710 kg 的雲霄飛車由一個巨大的彈簧發射出去,沿著光滑的翻筋斗迴圈軌道移動,如下圖所示,彈簧之彈性常數為$k = 34\,kN/m$,迴圈的半徑為 7.5 m,求使雲霄飛車環繞迴圈彈簧所需的最小壓縮距離。(A) 2.35 cm, (B) 2.50 cm, (C) 2.62 cm, (D) 2.77 cm.

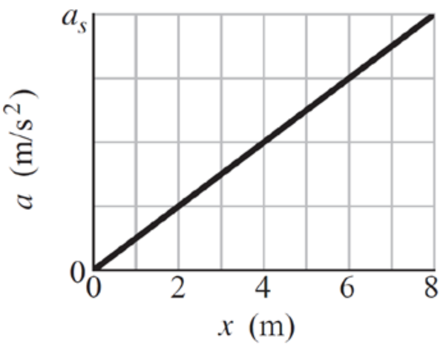


20. 下圖中,在非常靠近兩個氫原子位能曲線底部的部分,位能可以用近似值 $U = U_0 + A(x - x_0)^2$ 來表示,其中 $U_0 = -0.760\,aJ$, $A = 286\,aJ/nm^2$, $x_0 = 0.0741\,nm$, 後者為平衡間距。如果總能量為 -0.717 aJ,求原子間距的範圍. (A) 自 0.0689 nm 至 0.0793 nm, (B) 自 0.0618 nm 至 0.0864 nm, (C) 自 0.0655 nm 至 0.0827 nm, (D) 自 0.0555 nm 至 0.0927 nm



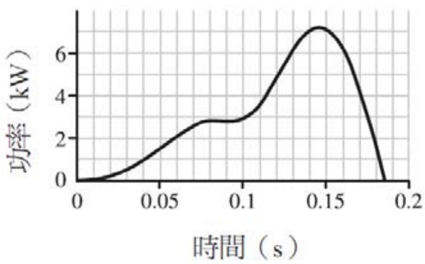
21. 有一質子(質量$m = 1.67 \times 10^{-27}kg$)在線性加速器中，被直線加速到 $3.6\times 10^{15} m/s^2$ 的速度。若質子的初速為 $2.4 \times 10^7 m/s$ 且運動了 3.5 cm 後，則 (1) 該質子的速率為多少？(A) $2.9 \times 10^7 m/s$, (B) $5.8 \times 10^7 m/s$, (C) $2.9 \times 10^8 m/s$, (D) $5.8 \times 10^8 m/s$; (2) 質子動能的增量為何？ (A) $1.1 \times 10^{-13}\,J$, (B) $2.1 \times 10^{-13}\,J$, (C) $4.2\times 10^{-13}\,J$, (D) $6.3 \times 10^{-13}\,J$.

22. 有一質量 15kg 的磚塊沿著 x 軸運動,其加速度是位置的函數,如下圖所示,此圖垂直軸的刻度設定為$a_s = 24.0\,m/s^2$。當磚塊由$x = 0$移動至$x = 8.0$ m時，產生此加速度的力作用於磚塊上的淨功為何？ (A) $1.0 \times 10^3\,J$, (B) $1.2 \times 10^3\,J$, (C) $1.4 \times 10^3\,J$, (D) $1.6 \times 10^3\,J$



23. 將一根 0.50 kg 的香蕉以 4.00 m/s 初速垂直往上拋出，它到達的最大高度是 0.80 m。在香蕉上升期間，空氣阻力將導致香蕉－地球系統的力學能改變是多少？ (A) 0.02 J, (B) 0.04 J, (C) 0.06 J, (D) 0.08 J
24. 一顆 1.50 kg 雪球以 20.0 m/s 的初速斜向射出，其方向為水平向上 34.0°角。試求：(1) 其初始動能為多少？(A) 100 J, (B) 200 J, (C) 300 J, (D) 400 J; (2) 當雪球從拋射點移至最高點時，雪球－地球系統的重力位能改變量是多少？(A) 63.8 J, (B) 73.8 J, (C) 83.8 J, (D) 93.8 J; (3) 雪球的最大高度是多少? (A) 6.38 m, (B) 9.57 m, (C) 12.76 m, (D) 19.04 m.
25. 工人推動一個 23 kg 的木箱沿地板等速前進 8.4m，此力為水平向下 32°角。若木箱與地板的動摩擦係數為 0.20，則 (1) 工人出力所作的功為何？ (1) 435 J, (2) 653 J, (3) 870 J, (D) 1306 J; (2) 木箱－地球系統的熱能增量為何？ (1) 435 J, (2) 653 J, (3) 870 J, (D) 1306 J.

1. You slide a box of books at constant speed up a 30° ramp, applying a force of 200 N directed up the slope. The coefficient of sliding friction is 0.18. (1) How much work have you done when the box has risen 1 m vertically? (A) 200 J, (B) 400 J, (C) 600 J, (D) 800 J; (2) What's the mass of the box? (A) 15.5 kg, (B) 31.0 kg, (C) 46.5 kg, (D) 62.0 kg.
2. How much work does a force $\vec{F} = (67\hat{\imath} + 23\hat{\jmath} + 55\hat{k})$N do as it acts on a body moving in a straight line from $\vec{r_1} = (16\hat{\imath} + 31\hat{\jmath})$m to $\vec{r_2} = (21\hat{\imath} + 10\hat{\jmath} + 14\hat{k})$m ? (A) 335 J, (B) 483 J, (C) 622 J, (D) 770 J.
3. At what rate can a half-horsepower (1 hp = 746 W) well pump deliver water to a tank 80 m above the water level in the well? (Give your answer in kg/s). (A) 373 kg/s, (B) 38.1 kg/s, (C) 0.47 kg/s, (D) 0.94 kg/s.
4. A 1400-kg car ascends a mountain road at a steady 60 km/h, against a 550-N force of air resistance. If the engine supplies energy to the drive wheels at the rate of 38 kW, what's the slope angle of the road? (A) 7.2°, (B) 14.4°, (C) 21.6°, (D) 28.8°
5. The energy in a batted baseball comes from the power delivered while the bat is in contact with tha ball. The most powerful hitters can supply some 10 horsepower (1 hp = 746 W) during the brief contact time, propelling the ball to over 100 miles per hour. The figure shows data taken from a particular hit, giving the power the bat delivers to the ballas a function of time.



(1) Which of the following is greatest at the peak of the curve?
   (A) The ball's kinetic energy
   (B) The ball's speed
   (C) The total work the bat has done on the ball
   (D) The rate which the best supplies energy to the ball
(2) The ball has its maximum speed at about
   (A) 85 ms
   (B) 145 ms
   (C) 185 ms
   (D) Whenever the force is greatest
(3) As a result of being hit, the ball's kinetic energy increases by about
   (A) 550 J

(B) 1.3 kJ

(C) 7.0 kJ

(D) You can't tell because you don't know its speed coming from the pitcher.

(4) The force on the ball is greatest approximately

(A) at 185 ms

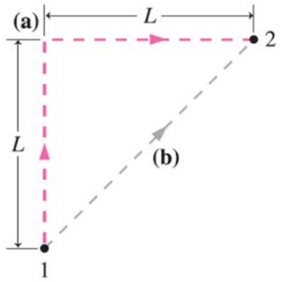
(B) before the peak in the figure

(C) at the peak in the figure

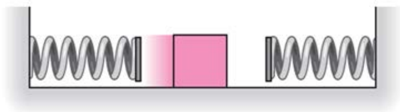
(D) after the peak in the figure but before 185 ms

6. After a tornado, a 0.5-g drinking straw was found embedded 4.5 cm in a tree. Subsequent measurements showed that the tree exerted a stopping force of 70 N on the straw. What was the straw's speed? (A) 27.5 m/s, (B) 55 m/s, (C) 82.5 m/s, (D) 110 m/s.
7. Workers pushing a 180-kg trunk across a level floor encounter a 10-m-long region where the floor becomes increasingly rough. The coefficient of kinetic friction here is given by $\mu_k = \mu_0 + ax^2$, where $\mu_0 = 0.17$, $a = 0.0062m^{-2}$, and x is the distance from the beginning of the rough region. How much work does it take to push the trunk across the region? (A) 6.6 kJ, (B) 9.9 kJ, (C) 13.2 kJ, (D) 19.8 kJ.
8. A tugboat pushes a cruise ship with force $\vec{F} = (1.2\hat{\imath} + 2.3\hat{\jmath})$MN, moving the ship along a straight path with displacement $\Delta\vec{r} = (380\hat{\imath} + 460\hat{\jmath})$m, Find (1) the work done b the tugboat: (A) 755 MJ, (B) 1510 MJ, (C) 2265 MJ, (D) 3120 MJ; and (2) find the angle between the force and displacement: (A) 12°, (B) 24°, (C) 36°, (D) 48°.
9. In midday sunshine, solar energy strikes Earth at the rate of about 1 kW/m^2. How long would it take a perfectly efficient solar collector of 15-m^2 area to collect 40 kWh of energy? (Note: This is roughly the energy content of a gallon of gasoline.) (A) 1.6h, (B) 2.0h, (C) 2.4h, (D) 2.7h
10. An elastic cord used in bungee jumping is normally 11 m long and has spring constant k = 250 N/m. At the lowest point in a jump, the cord length has doubled. (1) How much work has been done on the cord? (A) 5kJ, (B) 10 kJ, (C) 15 kJ, (D) 20 kJ. (2) As the cord stretches its final meter and first meter, respectively, what's the ratio of the work done between them? (A) 11, (B) 21, (C) 31, (D) 41.
11. (1) Determine the work you would have to do to move a block of mass m from point 1 to point 2 at constant speed over the two paths shown in the figure. The coefficient of friction has the constant value μ over the surface. Note: the diagram lies in a horizontal plane. (A) $W_a = -2\mu_k mgL$, $W_b = -\sqrt{2}\mu_k mgL$, (B) $W_a = -mgL$, $W_b = -mgL$, (C) $W_a = -\sqrt{2}\mu_k mgL$, $W_b = -2\mu_k mgL$, (D) $W_a = mgL$, $W_b = mgL$. (2) Now take the figure to lie in a vertical plane, and find the work done by the gravitational force as an object moves from point 1 to

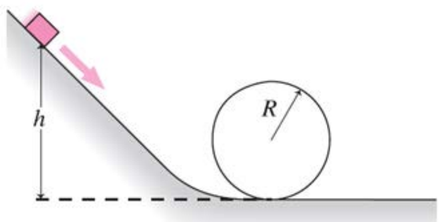
point 2 over each of the paths shown. (A) $W_a = -2\mu_k mgL,\ W_b = -\sqrt{2}\mu_k mgL$, (B) $W_a = -mgL,\ W_b = -mgL$, (C) $W_a = -\sqrt{2}\mu_k mgL,\ W_b = -2\mu_k mgL$, (D) $W_a = mgL,\ W_b = mgL$.



12. A particle slides back and forth on a frictionless track whose height as a function of horizontal position x is $y = ax^2$, where a = 0.95m^{-1}. If the particle's maximum speed is 9.2 m/s, find its turning points. (A) ± 0.55 m, (B) ± 1.1 m, (C) ± 1.7 m, (D) ± 2.1 m
13. Popes used in rock climbing are "springy" so that they cushion a fall. A particular rope exerts a force $\mathrm{F} = -\mathrm{kx} + \mathrm{b}x^2$, where $\mathrm{k} = 223\ N/m,\ \mathrm{b} = 4.10\ N/m^2$, and x is the stretch. Find the potential energy stored in this rope when it's been stretched 2.62 m, taking U = 0 at x = 0. (A) 185 J, (B) 371 J, (C) 556 J, (D) 741 J.
14. A 100-g block slides back and forth on a frictionless surface between teo springs, as shown in the figure. The left-hand spring has k = 110 N/m and its maximum compression is 20 cm. The right-hand spring has k = 240 N/m. Find (1) the maximum compression of the right-hand spring: (A) -7.0 cm, (B) -10.5 cm, (C) -13.5 cm, (D) -16.0 cm; and (2) find the speed of the block as it moves between the springs: (A) ± 6.63 m/s, (B) ± 8.63 m/s, (C) ± 10.63 m/s, (D) ± 12.63 m/s

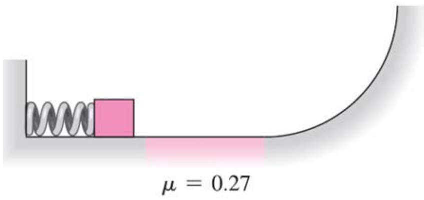


15. A lock slides on a frictionless loop-the-loop track shown in the figure. Find the minimum height h at which it can start from rest and still make it around the loop. (A) 3R/2, (B) 2R, (C) 5R/2, (D) 3R.

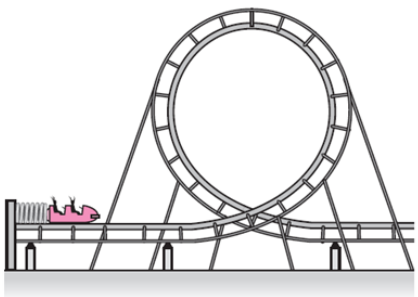


16. A mass m is dropped from height h above the top of a spring of constant k mounted vertically on the floor. Find the spring's maximum compression is (A) $(2mg/k)(1+\sqrt{1+kh/mg})$, (B) $(2mg/k)(1+\sqrt{1+2kh/mg})$, (C) $(mg/k)(1+\sqrt{1+kh/mg})$, (D) $(mg/k)(1+\sqrt{1+2kh/mg})$.

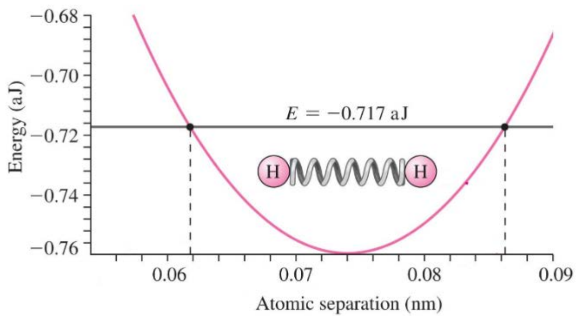
17. A skier starts down a frictionless 34° slpoe. After a vertical drop of 24 m, the slope temporarily levels out and then slopes down at 16°, dropping an additional 38 m vertically before leveling out again. (1) Find the skier's speed on the two level stretches. (A) $16.4\,\mathrm{m/s},\ 21.7\,\mathrm{m/s}$, (B) $21.7\,\mathrm{m/s},\ 34.9\,\mathrm{m/s}$, (C) $20\,\mathrm{m/s},\ 30\,\mathrm{m/s}$, (D) $30\,\mathrm{m/s},\ 40\,\mathrm{m/s}$ (2) Repeat problem (1) for the case when the coefficient of kinetic friction on both slopes is 0.11, while the level stretches remain frictionless. (A) $16.4\,\mathrm{m/s},\ 21.7\ \mathrm{m/s}$, (B) $21.7\,\mathrm{m/s},\ 34.9\,\mathrm{m/s}$, (C) $20\,\mathrm{m/s},\ 30\,\mathrm{m/s}$, (D) $30\,\mathrm{m/s},\ 40\,\mathrm{m/s}$
18. A 182-g block is launched by compressing a spring of constant $\mathrm{k} = 200\,N/m$ by 15 cm. The spring is mounted horizontally, and the surface directly under it is frictionless. Bur beyond the equilibrium position of the spring end, the surface has frictional coefficient μ = 0.27. This frictional surface extends 85 cm, followed by a frictionless curved rise, as shown in the figure. After it's launched, where does the block finally come to rest? (Measure from the left end of the frictional zone) (A) 22.9 cm, (C) 34.4 cm, (D) 45.8 cm, (D) 56.4 cm.



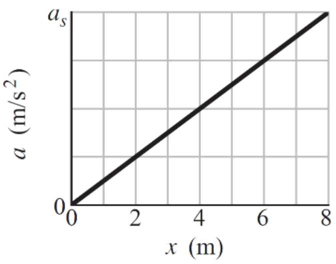
19. An 710-kg roller-coaster car is launched from a giant spring with $\mathrm{k} = 34\,kN/m$ into a frictionless loop-the-loop track of radius 7.5 m, as shown in the figure. What's the minimum spring compression that ensure the car stays on the track. (A) 2.35 cm, (B) 2.50 cm, (C) 2.62 cm, (D) 2.77 cm.



20. Very near the bottom of the potential well in the figure, the potential energy of the two-atom system given approximately by $\mathrm{U} = U_0 + A(x - x_0)^2$, where $U_0 = -0.760\,aJ$, $A = 286\,aJ/nm^2$, and $x_0 = 0.0741\,nm$ is the equilibrium separation. What range of atomic separations is allowed if the total energy is -0.717 aJ? (A) from 0.0689 nm to 0.0793 nm, (B) from 0.0618 nm to 0.0864 nm, (C) from 0.0655 nm to 0.0827 nm, (D) from 0.0555 nm to 0.0927 nm



21. A proton (mass m = 1.67 × 10^{-27} kg) is being accelerated along a straight line at 3.6 × 10^{15} m/s^2 in a machine. If the proton has an initial speed of 2.4 × 10^7 m/s and travels 3.5 cm, (1) what is its speed? (A) 2.9 × 10^7m/s, (B) 5.8 × 10^7m/s, (C) 2.9 × 10^8m/s, (D) 5.8 × 10^8m/s; and (2) what is the increase in its kinetic energy? (A) $1.1 \times 10^{-13}\ J$, (B) $2.1 \times 10^{-13}\ J$, (C) $4.2\times 10^{-13}\ J$, (D) $6.3 \times 10^{-13}\ J$.
22. A 15 kg brick moves along an x axis. Its acceleration as a function of its position is shown in the figre. The scale of the figure's vertical axis is set by as = 24.0 m/s^2. What is the net work performed on the brick by the force causing the acceleration as the brick moves from x = 0 to x = 8.0 m? (A) $1.0 \times 10^3\ J$, (B) $1.2 \times 10^3\ J$, (C) $1.4 \times 10^3\ J$, (D) $1.6 \times 10^3\ J$



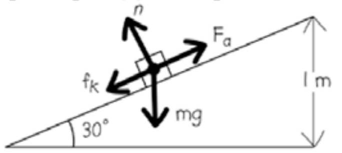
23. A 0.50 kg banana is thrown directly upward with an initial speed of 4.00 m/s and reaches a maximum height of 0.80 m. What change does air drag cause in the mechanical energy of the banana–Earth system during the ascent? (A) 0.02 J, (B) 0.04 J, (C) 0.06 J, (D) 0.08 J
24. A 1.50 kg snowball is shot upward at an angle of 34.0° to the horizontal with an initial speed of 20.0 m/s. (1) What is its initial kinetic energy? (A) 100 J, (B) 200 J, (C) 300 J, (D) 400 J; (2) By how much does the gravitational potential energy of the snowball–Earth system change as the snowball moves from the launch point to the point of maximum height? (A) 63.8 J, (B) 73.8 J, (C) 83.8 J, (D) 93.8 J; (3) What is that maximum height? (A) 6.38 m, (B) 9.57 m, (C) 12.76 m, (D) 19.04 m.
25. A worker pushed a 23 kg block 8.4 m along a level floor at constant speed with a force directed 32° below the horizontal. If the coefficient of kinetic friction between block and floor was 0.20, (1) what was the work done by the worker's force? (1) 435 J, (2) 653 J, (3) 870 J, (D) 1306 J. and (2) what was the increase in thermal energy of the block–floor system? (1) 435 J, (2) 653 J, (3) 870 J, (D) 1306

J.

1. (1) B, (2) B.
2. C
3. C
4. A
5. (1) D, (2) C, (3) A, (4) B
6. D
7. A
8. (1) B, (2) A
9. D
10. (1) C, (2) B
11. (1) A (2) B
12. D
13. D
14. (1) C, (2) A
15. C
16. D
17. (1) B, (2) C
18. A
19. D
20. B
21. (1) A, (2) B
22. C
23. D
24. (1) C, (2) D, (3) A
25. (1) A, (2) A

1.

Interpret The problem is about calculating work, given force and displacement. The object of interest is the box, which is being pushed up a ramp. For part **(b)** of the problem, we consider the work-energy theorem.



Develop Make a free-body diagram of the box (see figure). Use Equation 6.5, $W = \vec{F}\cdot\Delta\vec{r}$, to calculate the work done in pushing the box up the ramp.

Evaluate **(a)** The box rises $\Delta y = 1$ m vertically. This means that the displacement up the ramp (parallel to the applied force) is

$$\Delta r = \frac{\Delta y}{\sin(\theta)} = \frac{1\text{ m}}{\sin(30°)} = 2\text{ m}$$

Therefore, the work done during this process is

$$W_{\text{app}} = \vec{F}_{\text{app}}\cdot\Delta\vec{r} = (200\text{ N})(2\text{ m})\cos(0°) = 400\text{ J}$$

because the angle between the applied force and the displacement vector is 0°.

(b) To find the mass, we first note that the work done by gravity is

$$W_g = \vec{F}_g\cdot\Delta\vec{r} = (-mg\hat{j})\cdot(\Delta x\hat{i} + \Delta y\hat{j}) = -mg\Delta y = -mg\Delta r\sin\theta$$

The work done by friction is

$$W_f = \vec{f}_k\cdot\Delta\vec{r} = -f_k\Delta r = -\mu_k n\,\Delta r = -\mu_k(mg\cos\theta)\Delta r$$

where in the last step we have used $n - mg\cos(\theta) = 0$, which results from applying Newton's second law to the box in the direction perpendicular to the incline. Because the speed of the box remains unchanged, the work-energy theorem $W = \Delta K$, says the total work must be zero:

$$W_{\text{Tot}} = W_{\text{app}} + W_g + W_f = 0$$

This implies

$$W_{\text{app}} = -W_g - W_f = mg\Delta r\sin\theta + \mu_k(mg\cos\theta)\Delta r = mg\Delta r(\sin\theta + \mu_k\cos\theta)$$

from which the mass is found to be

$$m = \frac{W_a}{g\Delta r(\sin\theta + \mu_k\cos\theta)} = \frac{F_a}{g(\sin\theta + \mu_k\cos\theta)} = \frac{200\text{ N}}{(9.8\text{ m/s}^2)[\sin(30°) + (0.18)\cos(30°)]}$$
$$= 31\text{ kg}$$

2.

Interpret This problem involves finding the work done by the given force vector that acts through the given displacement.

Develop Use the general form of the expression for work, Equation 6.5: $W = \vec{F}\cdot\Delta\vec{r}$, with $\vec{F} = 67\hat{i} + 23\hat{j} + 55\hat{k}$ N and

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (21-16)\hat{i} + (10-31)\hat{j} + (14-0)\hat{k}\text{ m}$$
$$= 5\hat{i} - 21\hat{j} + 14\hat{k}$$

Evaluate Inserting the given force and displacement into Equation 6.5 gives

$$W = (67\hat{i} + 23\hat{j} + 55\hat{k}\text{ N})\cdot(5\hat{i} - 21\hat{j} + 14\hat{k}\text{ m})\vec{F}\cdot\Delta\vec{r} = (335 - 483 + 770)\text{ Nm} = 622\text{ J}$$

3.

INTERPRET In this problem the pump (with a given power) is doing work against gravity to deliver water to a tank above the ground. The quantity of interest is the amount of water that the pump can deliver during a given time interval.

DEVELOP According to Equation 6.15, if the average power is $\bar{P}$, then the amount of work done over a period Δt is $\Delta W = \bar{P}\Delta t$. Because the work required to lift an object of mass m to a vertical height h is $W = mgh$, the rate at which the mass can be delivered is

$$\bar{P} = \frac{\Delta W}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)gh \quad \Rightarrow \quad \frac{\Delta m}{\Delta t} = \frac{\bar{P}}{gh}$$

In SI units, 1 hp = 746 W.

EVALUATE Using the expression above, we find the rate at which water is delivered to the tank to be

$$\frac{\Delta m}{\Delta t} = \frac{\bar{P}}{gh} = \frac{(0.5\text{ hp})(746\text{ W/hp})}{(9.8\text{ m/s}^2)(80\text{ m})} = 0.47\text{ kg/s}$$

4.

INTERPRET In this problem a constant average power is supplied to the car as it climbs a slope against the air resistance. We want to know the angle of the slope if the car is moving at a steady speed.

DEVELOP At constant velocity, there is no change in kinetic energy, so the net work done on the car is zero. Therefore, the power supplied by the engine equals the power expended against gravity and air resistance. The power can be found from Equation 6.19, $P = \vec{F}\cdot\vec{v}$.

EVALUATE Because gravity $m\vec{g}$ makes an angle of $\theta+90°$ with the velocity $\vec{v}$ (where θ is the angle of the slope with respect to the horizontal), the power expended against gravity is

$$P_g = m\vec{g}\cdot\vec{v} = mgv\cos(\theta+90°) = -mgv\sin(\theta)$$

Similarly, the air resistance makes an angle of 180° to the velocity, so

$$P_{air} = \vec{F}_{air}\cdot\vec{v} = F_{air}v\cos(180°) = -F_{air}v$$

In SI units, v = 60 km/h = 16.7 m/s. Because the car moves at a constant speed, $P_{Tot} = P_{car} + P_g + P_{air} = 0$, or

$$P_{car} = -P_g - P_{air} = mgv\sin\theta + F_{air}v$$

Solving the equation gives

$$\theta = \text{asin}\left(\frac{P_{car} - F_{air}v}{mgv}\right) = \text{asin}\left[\frac{38000\text{ W} - (550\text{ N})(16.7\text{ m/s})}{(1400\text{ kg})(9.8\text{ m/s}^2)(16.7\text{ m/s})}\right] = 7.2°$$

5.

(1)

INTERPRET We're asked to analyze a graph of the power a bat imparts on a ball as a function of time.

DEVELOP The power, by definition is the rate at which the bat supplies energy to the ball.

EVALUATE The peak in the power is where the bat is delivering energy to the ball at the greatest rate. The answer is **(c)**.

(2)

INTERPRET We're asked to analyze a graph of the power a bat imparts on a ball as a function of time.

DEVELOP As argued in the previous problem, the speed continues to increase as long as the power is non-zero.

EVALUATE The speed will reach its maximum at the end of the hit, which occurs around 0.185 s on the graph. The answer is **(c)**.

(3)

INTERPRET We're asked to analyze a graph of the power a bat imparts on a ball as a function of time.
DEVELOP The change in the kinetic energy is equal to the work done by the bat: $\Delta K = W = \int P\,dt$. We can estimate this integral by roughly determining the area under the curve in the graph.
EVALUATE Each square in the grid has an area of $\Delta W = 1\text{ kW}\cdot 0.01\text{ s} = 10\text{ J}$. There are roughly 55 squares under the curve in the graph, so the total work done is 550 J, which is also the increase in the kinetic energy.
The answer is **(a)**.

(4)

INTERPRET We're asked to analyze a graph of the power a bat imparts on a ball as a function of time.
DEVELOP We can assume that the force provided by the bat and the velocity of the ball are parallel. Therefore, the bat force is given by: $F = P/v$. The power is maximum at the peak in the graph, P_{pk}, whereas the velocity constantly increases while the ball and bat are in contact (recall Problem 6.91).
EVALUATE We can rule out answer **(a)**, since the power is zero there, which implies the force is too. Near the peak, the power is not changing much (the derivative with respect to time is zero at the maximum). Therefore, at a point slightly before the peak, the power is essentially the same, but the velocity is smaller by some amount we will call Δv. The force at a point before the peak can be approximated as:

$$F_{\text{before}} = \frac{P_{\text{before}}}{v_{\text{before}}} \approx \frac{P_{\text{pk}}}{v_{\text{pk}} - \Delta v} \approx \frac{P_{\text{pk}}}{v_{\text{pk}}}\left(1 + \frac{\Delta v}{v_{\text{pk}}}\right) > F_{\text{pk}}$$

where we have used the binomial approximation from Appendix A: $(1-x)^{-1} \approx 1 + x$ for $x \ll 1$. By a similar argument, $F_{\text{after}} < F_{\text{pk}}$, so the force is greatest just before the peak.
The answer is **(c)**.
ASSESS One might question the reasoning above. If the velocity were changing more slowly than the power near the peak, then the force would be maximum at the peak, not before. However, we can show that this leads to a contradiction. The derivative of the force with respect to time is zero when the force is maximum:

$$\frac{dF}{dt} = \frac{d}{dt}\left[\frac{P}{v}\right] = \frac{1}{v}\frac{dP}{dt} - \frac{P}{v^2}\frac{dv}{dt} = 0$$

Assuming the maximum force occurs at the peak, then the derivative of the power would also be zero $(dP/dt = 0)$, since the peak is a maximum of the power as well. The equation above reduces to $dv/dt = 0$, which implies zero acceleration, zero force. But that contradicts the assumption that the peak is a maximum of the force. In conclusion, the maximum force has to occur before the peak.

6.

INTERPRET This problem involves work and the work-energy theorem. Given a force acting on an object and the distance over which the force acts, we are asked to find the initial velocity of the object.
DEVELOP The work-energy theorem, Equation 6.14 ($W_{\text{net}} = \Delta K$) tells us that the net work done on the straw is its change in kinetic energy, which involves the straw's initial speed. Because the stopping force acts in the same direction as the straw's displacement in the tree (i.e., it's a one-dimensional problem), and assuming the stopping force is constant, we can use Equation 6.1, $W = F_x\Delta x$ to find the net work done on the straw by the tree. Because the force of the tree acts to oppose the displacement of the straw, the work is negative : $W = -F_x x$, where $x = 4.5$ cm. Equating this to the change in kinetic energy by the work-energy theorem allows us to find the initial velocity of the straw.
EVALUATE Equating the work done by the tree to the change in the straw's kinetic energy, then solving for the initial speed of the straw gives

$$W_{\text{net}} = -Fx = \frac{m}{2}\left(\overbrace{v_2^2}^{=0} - v_1^2\right)$$

$$v_1 = \pm\sqrt{\frac{2Fx}{m}} = \sqrt{\frac{2(70\text{ N})(0.045)}{0.5\times10^{-3}\text{ kg}}} = 110\text{ m/s}$$

to two significant figures. Because the plus/minus sign simply indicates an initial velocity to the left or to the right, we have arbitrarily chosen the positive sign.

7.

題意：題目問的是工人推動行李箱所做的功。若要以等速度移動行李箱，工人必須施加與摩擦力相等的作用力，由於摩擦力與位置有關，所以我們要計算變化力所做的功。

思考：圖 6.13 顯示作用力相對於位置的曲線圖，所以我們要用方程式 $W=\int_{x_1}^{x_2} F(x)\,dx$ (6.8) 做積分。摩擦力可以用方程式 $f_k=\mu_k n$ (5.3) 計算，對水平地板而言，正向力的大小為重量 mg，因此方程式 6.8 變成

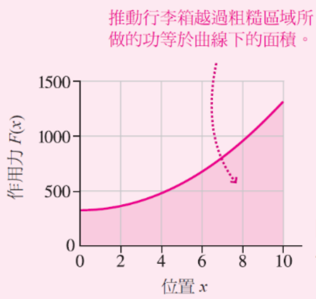


圖 6.13　例題 6.5 之作用力相對於位置的曲線圖。

$$W=\int_{x_1}^{x_2}\mu_k mg\,dx=\int_{x_1}^{x_2} mg(\mu_0+ax^2)\,dx$$ 。

解答：我們用方程式 6.9 計算積分。積分有兩個部分：一個是 dx 另一個是 x^2dx，從方程式 6.9 可知前者的積分為 x，後者為 $x^3/3$，因此結果為

$$W=\int_{x_1}^{x_2} mg(\mu_0+ax^2)\,dx=mg(\mu_0x+\tfrac{1}{3}ax^3)\Big|_{x_1}^{x_2}$$
$$=mg\left[(\mu_0x_2+\tfrac{1}{3}ax_2^3)-(\mu_0x_1+\tfrac{1}{3}ax_1^3)\right]$$

將 μ_0、a、m 以及 $g=9.8\ \mathrm{m/s^2}$ 的值代入，積分的上下限為粗糙區域的邊界也就是 $x_1=0$ 與 $x_2=10$ m，我們得到的答案為 6.6 kJ。

說明：這個答案合理嗎？從圖 6.13 可知最大作用力的大小約為 1.3 kN，如果這個作用力施加於整個 10 m 的範圍，所做的功將會是 13 kN。但因為動摩擦係數變化的關係，一開始的作用力很小，從圖 6.13 可知曲線下的面積約佔整個矩形的一半，因此我們的答案 6.6 kJ 是合理的。

8.

某拖船施加作用力 $\vec{F}=(1.2\hat{\imath}+2.3\hat{\jmath})$MN，沿一條直線的航道拉一艘遊艇行進一個位移 $\Delta\vec{r}=(380\hat{\imath}+460\hat{\jmath})m$。求 (a) 拖船所做的功；(b) 作用力與位移的夾角。

題意：(a) 的部分為從作用力與位移來計算功，作用力與位移都是以單位向量的形式來表示。(b) 的部分比較不明顯，但是因為功的計算牽涉到作用力與位移的夾角，所以 (a) 部分的計算結果可以用來計算 (b) 的部分。

思考：在圖 6.7 中畫出作用力與位移兩個向量，這可以用來檢查我們最後的答案。(a) 部分可以用方程式 6.5（$W=\vec{F}\cdot\Delta\vec{r}$）與純量積方程式 6.4 來計算，這樣

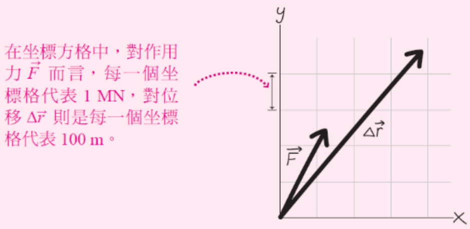


圖 6.7　例題 6.3 的向量圖。

就可以計算所要求的功 W。另外從向量 $\vec{F}$ 與 $\Delta\vec{r}$ 可以求作用力與位移的大小，有了功與向量的大小之後，接著可以用方程式 6.3 計算未知的夾角 θ，也就是 (b) 部分所要求的。

解答：對 (a) 部分我們用方程式 6.5 與 6.4 計算：

$$W=\vec{F}\cdot\Delta\vec{r}=F_x\,\Delta x+F_y\,\Delta y$$
$$=(1.2\ \mathrm{MN})(380\ \mathrm{m})+(2.3\ \mathrm{MN})(460\ \mathrm{m})=1510\ \mathrm{MJ}$$

在上面的方程式中，第一個等號來自方程式 6.5，第二個則是 6.4 純量積的計算，Δx 與 Δy 分別為 $\Delta\vec{r}$ 的 x 與 y 分量。現在已經計算了功，接著就可以計算角度。向量的大小用方程式 3.1 也就是畢氏定理計算：

$$F=\sqrt{F_x^2+F_y^2}=\sqrt{(1.2\ \mathrm{MN})^2+(2.3\ \mathrm{MN})^2}=2.59\ \mathrm{MN}$$

類似的計算可以得到 $\Delta r=597$ m。現在解方程式 6.3 可得 θ：

$$\theta=\cos^{-1}\left(\frac{W}{F\,\Delta r}\right)=\cos^{-1}\left(\frac{1510\ \mathrm{MJ}}{(2.59\ \mathrm{MN})(597\ \mathrm{m})}\right)=12^\circ$$

說明：這個小的角度與圖 6.7 所顯示者是相符合的，這相當合乎物理的道理：拖船最有效的推動方向應該與遊艇要前進的方向一致。請注意最後一個計算中單位的部分：MJ 在分子，MN·m 在分母，由於 1 N·m = 1 J，所以分母也是 MJ，因而符合餘弦的值是無因次的。

9.

INTERPRET　This problem is an exercise in converting from power to energy. We are to find the area needed to collect a given amount of energy given the parameters of solar radiation reaching the Earth's surface.

DEVELOP　If we multiply the power density hitting the surface of the Earth (1 kW/m^2) by the surface area (m^2) of our perfectly efficient solar collector, we get power (kW). This can be seen by dimensional analysis:

$$\bar{P} = \left(\frac{\text{kW}}{\cancel{\text{m}}^2}\right)\cancel{\text{m}}^2 = \text{kW}$$

The relationship between average power and time is given by Equation 6.14, $\bar{P} = \Delta W/\Delta t$, which we can use to solve this problem, given that the energy desired is $\Delta W = 40$ kW·h.

Evaluate The time it takes to collect $\Delta W = 40$ kW·h is thus

$$40\,\text{kW}\cdot\text{h} = \bar{P}\Delta t = \left(1\,\text{kW/m}^2\right)\left(15\,\text{m}^2\right)\Delta t$$

$$\Delta t = \frac{40\,\cancel{\text{kW}}\cdot\text{h}}{\left(1\,\cancel{\text{kW}}/\cancel{\text{m}}^2\right)\left(15\,\cancel{\text{m}}^2\right)} = 2.7\,\text{h}$$

10.

高空彈跳所使用之彈力繩索的長度通常是 11 m，彈性常數則為 $k = 250$ N/m，如果彈跳者抵達下降之最底端時，繩索長度變為原來的兩倍。求作用於繩索的功？

題意：彈性繩索就像彈簧一樣，因為題目給了我們彈性常數，因此本題目要計算伸展彈簧所做的功，由於 11 m 的繩索最後伸展為原來長度的兩倍，因此它伸長了 11 m。

思考：方程式 6.10 代表彈簧從未伸展的狀態到伸長 x 時外力所做的功。

解答：從方程式 6.10 可得

$$W = \tfrac{1}{2}kx^2 = (\tfrac{1}{2})(250\,\text{N/m})(11\,\text{m})^2 = 15\,\text{kN·m} = 15\,\text{kJ}$$

說明：你在後面的課文中很快會看到，這個數值相當於一個 70 kg 的人從繩索固定點自由下降，直到繩索全長伸展為 22 m 所做的功，在下一章我們會瞭解到這個相同的結果並非偶然的。

在例題 6.4 中，使繩索伸長最後 1 公尺所做的功，比起最初伸長 1 公尺大還是小？還是相等？

解答：我們要比較使彈性繩索伸長最初與最後 1 公尺，兩者間做功的差異。我們知道功等於作用力－距離曲線圖下的面積，作用力－距離曲線圖如圖 6.12 所示，圖中並將最初與最後 1 公尺的面積標示出來。

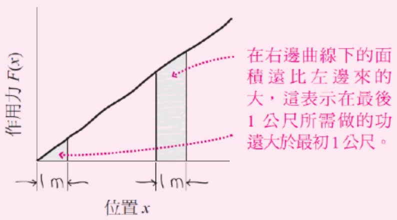


圖 6.12　觀念題 6.11。

說明：有道理！一旦繩索伸長了 10 m 它已經有較大的彈力，因此要再繼續伸長會比較困難，所以對最後 1 公尺就需要做很大的功。最初 1 公尺繩索的彈力很小，所需做的功自然很小。

相關計算　計算外力對繩索最初與最後 1 公尺所做的功，並比較之。

解答：我們可以用方程式 6.10 計算，但是對最初 1 公尺而言，要將原來的上下限 x 與 0 改成 1 m 與 0，對最後 1 公尺則要改成 11 m 與 10 m。計算的結果為 125 J 與 2.6 kJ。外力對伸展繩索最後 1 公尺所做的功超過最初 1 公尺的 20 倍！

11.

Interpret In this problem we want to find the work done by the frictional force in moving a block from one point to another over two different paths. Friction is not a conservative force, so mechanical energy is not conserved.

Develop Figure 7.15 is a plan view of the horizontal surface over which the block is moved, showing the paths **(a)** and **(b)**. The force of friction is $f_k = \mu_k n = \mu_k mg$ (see Equation 5.3) and is directed opposite to the displacement. Because f_k is constant, we use Equation 6.11,

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

Because the friction force is directed opposite to the displacement $d\vec{r}$, the scalar product introduces a negative sign. For path **(a)**, Equation 6.11 takes the form

$$W_a = -\int_{x_1}^{x_2} f_k\, dx - \int_{y_1}^{y_2} f_k\, dy = -f_k\left(x_2 - x_1\right) - f_k\left(y_2 - y_1\right)$$

where $x_1 = 0$, $y_1 = 0$, $x_2 = L$, $y_2 = L$. For path **(b)**, we use radial coordinates, and Equation 6.11 takes the form

$$W_b = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = -f_k \Delta r$$

where $\Delta r = \sqrt{L^2 + L^2} = \sqrt{2}L$, and the scalar product gives the negative sign because friction always acts opposite to the displacement.

Evaluate The work done by friction along path **(a)** is thus

$$W_a = -\mu_k mg\left(2L\right)$$

The work done by friction along path **(b)** is

$$W_b = -\sqrt{2}\mu_k mgL$$

Interpret This problem involves calculating the work done by a conservative force (gravity) and comparing the result obtained for the work done over two different paths.

Develop Take the origin at point 1 in Fig. 7.15 with the x axis horizontal to the right and the y axis vertically upward. Use the same equation for work as we did in Problem 7. 10 (Equation 6.11), but this time the force involved is the force of gravity: $\vec{F}_g = -mg\hat{j}$. For path **(a)**, we use Cartesian coordinates, so $d\vec{r} = dx\hat{i} + dy\hat{j}$. Inserting $\vec{F}_g$ into Equation 6.11 for path **(a)** thus gives

$$W_a = -\overbrace{\int_{x_1}^{x_2}\left(mg\hat{j}\right)\cdot dx\hat{i}}^{=0} - \int_{y_1}^{y_2}\left(mg\hat{j}\right)\cdot dy\hat{j} = -mg\left(y_2 - y_1\right)$$

For path **(b)**, we will use radial coordinates, so Equation 6.11 takes the form

$$W_b = -\int_{r_1=0}^{r_2=\sqrt{2}L} mg\hat{j}\cdot d\vec{r} = -\int_{r_1=0}^{r_2=\sqrt{2}L} mg\cos\left(45^\circ\right)dr = -\frac{1}{\sqrt{2}}\int_{r_1=0}^{r_2=\sqrt{2}L} mg\,dr$$

Evaluate Inserting the initial and final positions into the expression for path **(a)** gives $W_a = -mgL$. For path **(b)**, we find

$$W_b = -\frac{mg}{\sqrt{2}}\left(r_2 - r_1\right) = -\frac{mg}{\sqrt{2}}\left(\sqrt{2}L - 0\right) = -mgL$$

12.

INTERPRET This problem involves conservative forces and conservation of total mechanical energy. At the maximum height of the particle, we know its kinetic energy is zero and its potential energy is maximum. We will define the zero of potential energy as the particle's lowest position, where its kinetic energy will be maximum. We are asked to find the turning point of the particle, which is the x position where the particle stops rising and begins to fall again.

DEVELOP The particle's trajectory is given by the formula $y = ax^2$, with $a = 0.95\ \text{m}^{-1}$. The particles potential energy is $U = mgy$, and its kinetic energy is $K = mv^2/2$. Conservation of total mechanical energy tells us that the sum of these two quantities is conserved, and we can find that constant because we are told that the maximum speed (i.e., maximum kinetic energy) is 9.2 m/s, which must occur at the point where the potential energy is minimum (i.e., $y = 0$). Thus, we have

$$\frac{1}{2}mv^2_{\max} = K_{\max}$$

where $K_{\max}$ is constant and is the total mechanical energy, which is conserved. We can insert this into the general expression for total mechanical energy to find the turning point of the particle, because we know that the particle kinetic energy will be zero at the turning point.

EVALUATE The total mechanical energy is

$$K_{\max} = U + K = mgy + \frac{1}{2}mv^2 = mg(ax^2) + \frac{1}{2}mv^2$$

At the turning point, v = 0, so we have

$$K_{\max} = \frac{1}{2}mv^2_{\max} = mg\left(ax^2_{\text{turn}}\right)$$

$$x_{\text{turn}} = \pm v_{\max}\sqrt{\frac{1}{2ga}} = \pm(9.2\ \text{m/s})\sqrt{\frac{1}{2(9.8\ \text{m/s}^2)(0.95\ \text{m}^{-1})}} = \pm 2.1\ \text{m}$$

13.

用來攀岩的繩索具有相當好的彈性，因此可以作為掉落時的緩衝裝置。有一款繩索之彈力為 $F = -kx + bx^2$，其中 $k = 223\ \text{N/m}$，$b = 4.10\ \text{N/m}^2$，x 為伸長量，如果它被拉長 2.62 m，求儲存於繩索之位能，取 $x = 0$ 時 $U = 0$。

題意： 與例題 7.2 一樣，這是有關彈性位能的計算。但本題目的彈力比 $F = -kx$ 複雜，我們沒有現成的位能公式可以計算。

思考： 因為繩索之張力隨著伸長量而變化，所以我們必須用積分來計算。由於外在作用力與位移有相同的方向，我們用方程式 $\Delta U = -\int_{x_1}^{x_2} F(x)\,dx$ (7.2a) 來計算。

解答： 將繩索的張力代入可得

$$U = -\int_{x_1}^{x_2} F(x)\,dx = -\int_0^x (-kx + bx^2)\,dx = \tfrac{1}{2}kx^2 - \tfrac{1}{3}bx^3\Big|_0^x$$
$$= \tfrac{1}{2}kx^2 - \tfrac{1}{3}bx^3$$
$$= (\tfrac{1}{2})(223\ \text{N/m})(2.62\ \text{m})^2 - (\tfrac{1}{3})(4.1\ \text{N/m}^2)(2.62\ \text{m})^3$$
$$= 741\ \text{J}$$

說明： 計算的結果比具有相同彈性常數之理想彈簧的位能 $U = \frac{1}{2}kx^2$ 少了大約 3% 的能量，這是由於額外的一項 $+bx^2$ 所產生的效果，其本身為正的符號減低了繩索的回復力，也降低了伸展彈簧所需要做的功。

14.

INTERPRET The two forces acting on the block are those applied by the springs, so they are conservative forces. In the absence of friction and air resistance, we can apply conservation of total mechanical energy.

DEVELOP When the left hand spring is at its maximum compression, the block is instantaneously motionless, so the total mechanical energy of the block/springs system is just the elastic potential energy of the left-hand spring, so $U_L^{Tot} = k_L x_L^2/2$. At the opposite end, the total mechanical energy is $U_R^{Tot} = k_R x_R^2/2$. Between the springs the total energy is just the kinetic energy, so $U_R^{Tot} = mv^2/2$. By conservation of total mechanical energy, we can equate all three energies to find the compression x_R of the right-hand spring and the speed *v* of the block between the springs.

EVALUATE **(a)** At the right-hand end, the spring compresses a distance

$$k_L x_L^2 = k_R x_R^2$$
$$x_R = \pm x_L \sqrt{\frac{k_L}{k_R}} = -(0.2\ \text{m})\sqrt{\frac{110\ \text{N/m}}{240\ \text{N/m}}} = -13.5\text{cm}$$

where we have chosen the negative sign because the right-hand spring compresses.

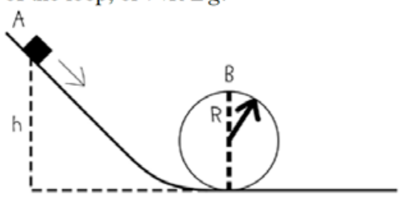
(b) The speed of the block between the springs is

$$\frac{k_L x_L^2}{2} = \frac{mv^2}{2}$$
$$v = \pm x_L \sqrt{\frac{k_L}{m}} = \pm(0.2\ \text{m})\sqrt{\frac{110\ \text{N/m}}{0.1\ \text{kg}}} = \pm 6.63\ \text{m/s}$$

15.

INTERPRET Because the track is frictionless (and we ignore air resistance), the only force acting on the block is gravity, which is a conservative force. Therefore, we can apply the conservation of total mechanical energy to this problem. We will choose the zero of gravitational potential energy to be the base of the loop.

DEVELOP Apply conservation of total mechanical energy, Equation 7.7 ($U_0 + K_0 = U + K$). The initial total mechanical energy is just the gravitational potential energy because the speed (i.e., kinetic energy) is zero at the start. Therefore, $U_0 = mgh$. The energy at the top of the loop is $U + K = 2mgR + mv^2/2$, where R is the radius of the loop (see figure below). For the block to stay on the track, the centripetal acceleration of the block must exceed the acceleration due to gravity at the top of the loop, or $v^2/R \geq g$.



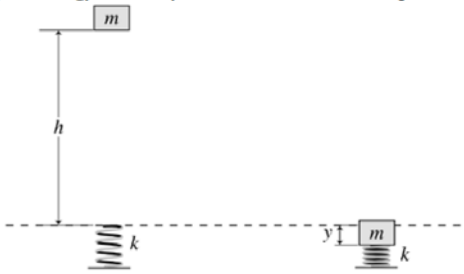
EVALUATE Equating the two expressions for total mechanical energy and using the minimum speed criterion gives

$$mgh = 2mgR + \frac{1}{2}mv^2$$
$$v^2 = 2gh - 4gR \geq gR$$
$$h \geq \frac{5}{2}R$$

16.

Interpret This problem involves the forces of gravity and of an elastic spring, both of which are conservative forces. Therefore, we can apply conservation of total mechanical energy. We take the zero of the gravitational potential energy the height of the spring in equilibrium.

Develop To apply conservation of total mechanical energy (Equation 7.7), we need to express the total mechanical energy for the block before it is dropped and when the spring is maximally compressed by the block (at which point the block is instantaneously motionless, see figure below). For the former, we have $U_0 + K_0 = mgh$. For the latter, we have $U + K = ky^2/2 - mgy$, where y is the distance from equilibrium that the spring is compressed.



Evaluate Equating the two expressions above for total mechanical energy and solving for the maximum spring compression y gives

$$mgh = \frac{1}{2}ky^2 - mgy$$

$$\left(\frac{k}{2}\right)y^2 + \left(-mg\right)y + \left(-mgh\right) = 0$$

$$y = \frac{mg \pm \sqrt{m^2g^2 + 2kmgh}}{k} = \frac{mg}{k}\left(1 + \sqrt{1 + 2kh/mg}\right)$$

17.

Interpret In this problem we are asked to find the speed of the skier at two different locations, given that the downward slope has a coefficient of friction $\mu_k = 0.11$. Because friction is a nonconservative force, we cannot apply conservation of total mechanical energy. Instead, we must use the concept of work done by a force combined with total mechanical energy.

Develop We find the work done by the friction force and subtract this work from the total energy to find the energy remaining after each slope The work done by friction skiing down a straight slope of length L is

$$W_f = -f_k L = -\mu_k nL = -\mu_k\left(mg\cos\theta\right)\left(\frac{h}{\sin\theta}\right) = -\mu_k mgh\cot\theta$$

where $h = L\sin\theta$ is the vertical drop of the slope. Conservation of energy applied between the start and the first level section now gives $\Delta K_{AB} + \Delta U_{AB} = W_{f,AB}$ or

$$\frac{1}{2}mv_B^2 = mg\left(y_A - y_B\right) - \mu_k mg\left(y_A - y_B\right)\cot\theta_{AB}$$

Similarly, for the motion between the top and the second level, we must include all the work done by friction, so

$$\Delta K_{AC} + \Delta U_{AC} = W_{f,AB} + W_{f,BC}$$

or

$$\frac{1}{2}mv_C^2 = mg\left(y_A - y_C\right) - \mu_k mg\left(y_A - y_B\right)\cot\theta_{AB} - \mu_k mg\left(y_B - y_C\right)\cot\theta_{BC}$$

Evaluate Solving the equation for v_B, we obtain

$$v_B = \sqrt{2g(y_A - y_B)(1-\mu_k \cot\theta_{AB})} = \sqrt{2(9.8\text{ m/s}^2)(25\text{ m})[1-0.11\cot(32°)]} = 20\text{ m/s}$$

Similarly, for v_C, we have

$$\begin{aligned} v_C &= \sqrt{2g\left[(y_A - y_C) - \mu_k(y_A - y_B)\cot\theta_{AB} - \mu_k(y_B - y_C)\cot\theta_{BC}\right]} \\ &= \sqrt{2(9.8\text{ m/s}^2)[63\text{ m} - (0.11)(25\text{ m})\cot(32°) - (0.11)(38\text{ m})\cot(20°)]} \\ &= 30\text{ m/s} \end{aligned}$$

ASSESS Let's consider the case where $\mu_k = 0$. In this limit, the results become

$$v_B = \sqrt{2g(y_A - y_B)} = \sqrt{2(9.8\text{ m/s}^2)(25\text{ m})} = 22\text{ m/s}$$
$$v_C = \sqrt{2g(y_A - y_C)} = \sqrt{2(9.8\text{ m/s}^2)(63\text{ m})} = 35\text{ m/s}$$

which are the same as the result of Problem 5.19 for the frictionless case.

18.

INTERPRET In this problem we want to find the final position of a block after being launched from a compressed spring. Its path involves a frictional surface followed by a frictionless curve. There forces acting on the block are conservative (gravity and the elastic force) and nonconservative (friction). We will define the block's initial position as the zero of gravitational potential energy.

DEVELOP The energy of the block when it first encounters friction is completely kinetic and, by conservation of total mechanical energy (Equation 7.7) it is equal to the initial elastic potential energy of the block/spring system:

$$K_0 = \frac{1}{2}kx^2$$

Upon crossing the friction zone, the work done by the friction is

$$W_{nc} = -\mu_k mgL$$

Depending on the ratio of $K_0/|W_{nc}|$, the block will move back and forth several times before losing all its energy and coming to rest.

EVALUATE Initially the block has an energy

$$K_0 = \frac{1}{2}kx^2 = \frac{1}{2}(200\text{ N/m})(0.15\text{ m})^2 = 2.25\text{ J}$$

The work done by the friction is

$$\Delta E = W_{nc} = -\mu_k mgL = -(0.27)(0.18\text{ kg})(9.8\text{ m/s}^2)(0.85\text{ m}) = -0.404\text{ J}$$

Because $K_0/|W_{nc}| = 5.27$, five complete crossings are made, leaving the block with energy $K = K_0 - 5|W_{nc}| = 0.109\text{J}$ on the curved side. This remaining energy is sufficient to move the block a distance

$$s = \frac{K}{\mu_k mg} = \frac{0.109\text{ J}}{(0.27)(0.18\text{ kg})(9.8\text{ m/s}^2)} = 0.229\text{ m}$$

so the block comes to rest 85 cm – 22.9 cm = 62.11 cm to the right of the beginning of the friction patch.

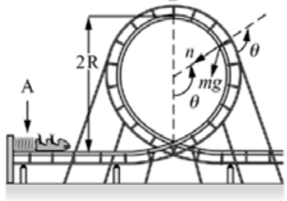
19.

INTERPRET The object of interest is the roller coaster that, after being launched from a compressed spring, moves along a frictionless circular loop. The physical quantity we are asked about is the minimum compression of the spring that allows the car to stay on the track. To find this, we will need to apply Newton's second law as well as conservation of mechanical energy.

DEVELOP If the car stays on the track, the normal force applied by the track must be greater than zero and the radial component of the cars acceleration is $a = v^2/R$. Applying Newton's second law to the roller coaster gives

$$n = \frac{mv^2}{R} + mg\cos\theta \geq 0 \quad \rightarrow \quad v^2 \geq -gR\cos\theta$$

The function $-\cos\theta$ is maximal at the top of the loop ($\theta = 180°$, see figure below), so $v_B^2 \geq gR$ is the condition for the car to stay on the track all the way around. This is the result obtained in Example 5.7.



With the minimum speed at point B determined, apply conservation of total mechanical energy (Equation 7.7), to find the minimum compression length of the spring.

EVALUATE In the absence of friction, conservation of total mechanical energy requires

$$K_A + U_A = K_B + U_B \quad \rightarrow \quad 0 + \frac{1}{2}kx^2 + mgy_A = \frac{1}{2}mv_B^2 + mgy_B$$

Solving for x, we obtain

$$x^2 = \frac{m}{k}\left[v_B^2 + 2g\left(y_B - y_A\right)\right] \geq \frac{5mgR}{k}$$

or

$$x \geq \pm\sqrt{\frac{5mgR}{k}} = \sqrt{\frac{5(710\text{ kg})(9.8\text{ m/s}^2)(7.5\text{ m})}{34{,}000\text{ N/m}}} = 2.77\text{ m}$$

20.

圖 7.11 中，在非常靠近兩個氫原子位能曲線底部的部分，位能可以用近似值 $U = U_0 + a(x - x_0)^2$ 來表示，其中 $U_0 = -0.760$ aJ，$a = 286$ aJ/nm^2，$x_0 = 0.0741$ nm，後者為平衡間距。如果總能量為 -0.717 aJ，求原子間距的範圍。

題意： 這裡所提到的分子能量具有奇怪的單位，也使題目看起來有點複雜，但其實它的內容與上面兩個圖 7.10 以及 7.11 所包含者是差不多的，因為我們要求的為轉折點，也就是代表總能量的水平線與位能曲線的相交點。如果單位看起來怪異，請記得公制中的字首（參閱書本最前面一頁的列表）是用來避免寫出 10 的大冪次，在這裡 1 aJ = 10^{-18} J，1 nm = 10^{-9} m。

思考： 圖 7.12 為題目中函數之位能曲線圖，圖中的水平直線代表總能量 E，轉折點為兩條曲線的相交點，也就是原子之間距，我們可以從圖中讀出來，或者使總能量等於位能並解代數方程式。

解答： 題目所給的位能與總能量，分別為 $U = U_0 + a(x - x_0)^2$ 與 E，因此轉折點可以從 $E = U_0 + a(x - x_0)^2$ 計算，我們可以直接計算用二次方程式的公式求 x，但是求 $x - x_0$ 值會比較容易：

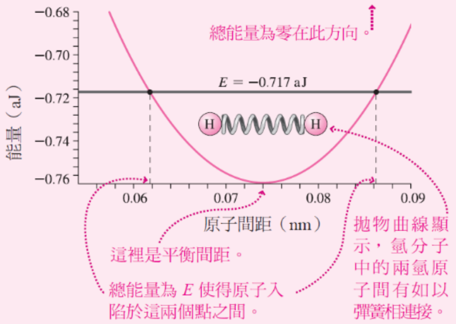


圖 7.12 分析氫分子。

$$x - x_0 = \pm\sqrt{\frac{E - U_0}{a}} = \pm\sqrt{\frac{-0.717\ \text{aJ} - (-0.760\ \text{aJ})}{286\ \text{aJ/nm}^2}}$$

$$= \pm 0.0123\ \text{nm}$$

因此轉折點滿足 $x_0 \pm 0.0123$ nm，也就是 0.0864 nm 與 0.0618 nm。

說明： 合理嗎？從圖 7.12 中可知，我們已經找到了兩個轉折點。位能曲線為拋物線（有如一個彈簧的位能 $U = \frac{1}{2}kx^2$）的事實，顯示氫分子大致上可以用兩個氫原子以一條彈簧相連接來作為模型。化學家經常使用此模型，對連接原子形成分子之化學鍵也會用到「彈性常數」的術語來形容。

21.

(a) From Table 2-1, we have $v^2 = v_0^2 + 2a\Delta x$. Thus,

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{\left(2.4\times10^7\ \text{m/s}\right)^2 + 2\ \left(3.6\times10^{15}\ \text{m/s}^2\right)\left(0.035\ \text{m}\right)} = 2.9\times10^7\ \text{m/s}.$$

(b) The initial kinetic energy is

$$K_i = \frac{1}{2}mv_0^2 = \frac{1}{2}\ \left(1.67\times10^{-27}\ \text{kg}\right)\left(2.4\times10^7\ \text{m/s}\right)^2 = 4.8\times10^{-13}\ \text{J}.$$

The final kinetic energy is

$$K_f = \frac{1}{2}mv^2 = \frac{1}{2}\ \left(1.67\times10^{-27}\ \text{kg}\right)\left(2.9\times10^7\ \text{m/s}\right)^2 = 6.9\times10^{-13}\ \text{J}.$$

The change in kinetic energy is $\Delta K = 6.9 \times 10^{-13}$ J – 4.8×10^{-13} J = 2.1×10^{-13} J.

22.

According to the graph the acceleration a varies linearly with the coordinate x. We may write $a = \alpha x$, where α is the slope of the graph. Numerically,

$$\alpha = \frac{24\ \text{m/s}^2}{8.0\ \text{m}} = 3.0\ \text{s}^{-2}.$$

The force on the brick is in the positive x direction and, according to Newton's second law, its magnitude is given by $F = ma = m\alpha x$. If x_f is the final coordinate, the work done by the force is

$$W = \int_0^{x_f} F\ dx = m\alpha \int_0^{x_f} x\ dx = \frac{m\alpha}{2} x_f^2 = \frac{(15\ \text{kg})(3.0\ \text{s}^{-2})}{2}(8.0\ \text{m})^2 = 1.4\times 10^3\ \text{J}.$$

23.

Eq. 8-33 gives $mgy_f = K_i + mgy_i - \Delta E_{\text{th}}$, or

$$(0.50\ \text{kg})(9.8\ \text{m/s}^2)(0.80\ \text{m}) = \frac{1}{2}\ (0.50\ \text{kg})(4.00\ \text{/s})^2 + (0.50\ \text{kg})(9.8\ \text{m/s}^2)(0) - \Delta E_{\text{th}}$$

which yields $\Delta E_{\text{th}} = 4.00\ \text{J} - 3.92\ \text{J} = 0.080\ \text{J}$.

24.

(a) The initial kinetic energy is $K_i = (1.5\ \text{kg})(20\ \text{m/s})^2 / 2 = 300\ \text{J}$.

(b) At the point of maximum height, the vertical component of velocity vanishes but the horizontal component remains what it was when it was "shot" (if we neglect air friction). Its kinetic energy at that moment is

$$K = \frac{1}{2}(1.5\ \text{kg})\left[(20\ \text{m/s})\cos 34°\right]^2 = 206\ \text{J}.$$

Thus, $\Delta U = K_i - K = 300\ \text{J} - 206\ \text{J} = 93.8\ \text{J}$.

(c) Since $\Delta U = mg\,\Delta y$, we obtain $\Delta y = \dfrac{94\ \text{J}}{(1.5\ \text{kg})(9.8\ \text{m/s}^2)} = 6.38\ \text{m}$.

25.

Since the velocity is constant, $\vec{a} = 0$ and the horizontal component of the worker's push $F \cos \theta$ (where $\theta = 32°$) must equal the friction force magnitude $f_k = \mu_k F_N$. Also, the vertical forces must cancel, giving:

$$F \cos \theta - \mu_k F_N = 0$$
$$F_N - F \sin \theta - mg = 0$$

which is solved to find $F = 61$ N.

(a) The work done on the block by the worker is, using Eq. 7-7,

$$W_F = Fd \cos \theta = (61 \text{ N})(8.4 \text{ m}) \cos(32^0) = 435 \text{ J}$$

(b) Since $f_k = \mu_k (mg + F \sin \theta)$, we find $\Delta E_{\text{th}} = f_k d = (52 \text{N})(8.4 \text{m}) = 436 \text{ J} \approx W_F$.