Lecture 5

Industrial Agglomeration with Production Externalities

Agglomeration of Economics

- Large Scale (Monopolistic Competition)
  - Migration (Krugman, JPE 1991)
  - Up-down Industry Linkage (Venables, IER 1996)
- Small Scale
  - Transportation Network Externality (Mori, JUE 1997, Fujita&Mori 2004)
  - Marshallian Externality
  - R&D (or Knowledge Externality) (Lucas & Rossi-Hansberg, Econometrica, 2002, Berliant, Peng and Wang, JET 2002)

Ch. 5 Industrial Agglomeration with Production Externalities

Referred Paper:

1. Berliant, Marcus, Shin-Kun Peng, and Ping Wang, 2002, "**Production Externalities and Urban Configuration**," ***Journal of Economic Theory*** 104, 275-303. (*Leading Article*)

2. Lucas, R.E., and Esteban Rossi-Hansberg (2002), "**On the Internal Structure of Cities**," ***Econometrica*** 70, 1445-1476.

- 2. Lucas, R. E., 2001, "Externalities and Cities," **Review of Economic Dynamics** 4, 245-274.

3. Lucas , R.E., and Esteban Rossi-Hansberg, 2002, " On the Internal Structure of Cities," **Econometrica** 70, 1445-1476.

5.1. Introduction

5.1-1 Theoretical Foundation

(1) Since the second half of the 1980s, the Marshallian externality has been commonly used as the primary driving force for city formation and industrial agglomeration.

(2) Jacobs (1969) claimed that ***uncompensated knowledge spillovers*** have played a central role in population agglomeration and thus in the generation of cities. (***The Economy of Cities)***

(3).***Uncompensated knowledge spillovers*** are intellectual gains by exchange of information for which no direct compensation to the producer of the knowledge is given or for which less compensation is given than the value of the knowledge.

(Marjolein C.J. Caniels, ***Knowledge Spillovers and Economic Growth, 2000).***

(4). Shell (1966,*AER*), Romer (1986, *JPE*) and Lucas (1988,*JME*) formalize this idea to create a revolution in their attempts to elaborate on the determinants of the endogenous rate of economic growth, giving birth to the so-called ***new growth theory***.

5.1-2 Empirical Literature

(1) Glaeser et al (1992, *JPE*)

- 170 USA Standard Metropolitan Area (SMA) cities, 1956-1987,
- Top-six two digit industries,
  - Knowledge spillover: Between > Within.

(2) Henderson et al (1995, *JPE*)

- 224 USA Standard Metropolitan Area (SMA) cities, 1970-1987,
  - Knowledge spillover:

    Capital goods industries: Within > Between,

    High-tech industries: both within and between are significant.

(3) Jaffe *et. al* (1993,*QJE*)

- Patent Data,
- USA Standard Metropolitan Area (SMA)

  950 (1975) → 4750 (1989),

  1450 (1980) → 5200 (1989),

- Knowledge spillover:

  They conclude that knowledge spillovers are geographically concentrated in the sense that patents are more likely to cite previous patents from the same area.

(4). Rauch (1993, *JUE*)

- 1980 USA SMA based Census data,
- 237 SMA cities,69910 individuals, 44758 households.
  - Cities with higher average level of human capital have higher wages and higher land rent.

(5). Ciccone and Hall (1996, *AER*)

- Doubling employment density induces to increase 6% labor productivity.
  - Geographical concentration is shown to improve productivity.

(6). Saxenian (1996): Regional Advantage

She make a formation of research parks, such as Boston’s Route 128 and California’s Silicon Valley [e.g., see a discussion by with respect to the importance of knowledge spillovers between (vertically) integrated firms.]

5.1-3 Urban (Spatial) Configuration Literature

(1) Fujita and Ogawa (1982, *RS&UE*)

They incorporate a locational potential function in which a weighted average of pair wise Euclidean distances between firms has a negative effect on firms' profit. Thus, this paper focuses on the externality of business agglomeration.

(2). Fujita and Krugman (1997, ***Regional Science: Perspectives for the Future***)

They consider monopolistic competition to examine the agglomeration (or externality) among firms.

(3). Fujita and Thisse (1986, *RES*)

They employ a Nash equilibrium concept with oligopoly to reexamine the spatial competition with consideration of land market, they obtained the various spatial configuration in contrast with pure spatial competition.

(4). Palivos and Wang (1996, *RS&UE*)

They consider knowledge spillovers in a monocentric city setting with endogenous growth. Their paper focuses on determining the optimal paths for output and population growth and the contrast between decentralized and socially optimal outcomes. They treat the monocentric urban configuration as exogenously given and assume unified household-firm units.

(5). Mai and Peng (1999, *RS&UE*)

They develop a model to incorporate the cooperation (centripetal) and competition (centrifugal) between firms, if

- the cooperation dominates competition: **Agglomeration**,
- the competition dominates cooperation: **Dispersed**.

5.1-4 Motivation: In this Chapter

(1) we explore this rich idea formally by extending the Romer (1986) model of positive externalities in production to an explicit spatial context.

(2) we examine the economic and geographical interplay between households and firms, and as a consequence, determines the equilibrium urban configuration endogenously.

5.2. The Model

(I) The Space setting and Assumptions

(1).Consider a linear city spread over a featureless "long-narrow" line represented by $\Omega \equiv [-1,1]$, with uniformly distributed land.

(2).There is a continuum of firms of mass M and a continuum of households of mass N (with N + M = 2, N > 0, M > 0).

(3). Each household occupies a unit density of land. For simplicity, we assume that there is an ***absentee landlord***, who owns all the land (of measure two) and consumes no land.

(4).The aggregate capital stock (of all firms located in the city, denoted $\bar{K}$) has a positive effect on the individual production of each firm.

In contrast with Romer (1986), we allow the magnitude of this positive externality to diminish with distance.

(II) Notations and Definitions

- Let *z* be the location index, $z \in \Omega \equiv [-1,1]$

- Denote the density of firms at location *z* under a particular urban configuration τ (to be determined in equilibrium) by $m_\tau(z)$,

- ***Mean location*** of firm sites as $\mu = \int_{z \in \Omega} z \cdot m_\tau(z) dz$ and

- The ***overall dispersion*** of firm sites as σ_{τ} ,

we require this overall dispersion index to be
(i) absolute (invariant to adding a constant to every firm's location) ,
(ii) decomposable (into subgroups with subgroup consistency),
(iii) symmetric (to the mean location).

The measures in the Kolm-Pollak class satisfy these properties [see Kolm (1976) and Pollak (1979)]. The simplest among these is an absolute deviation measure:

$$\sigma_{\tau} = (2/M) \int_{z \in \Omega} m_{\tau}(z) | z - \mu | dz$$

Denote the ***Degree of effectiveness of interactions*** as

$$Q(z) = 2 - (z - \mu)^2 - \varepsilon \sigma_\tau^2 > 0$$

It is to measure the ***degree of effectiveness of interactions*** between a particular firm z and the others in the linear city given a configuration of type τ, where $\varepsilon \in (0,1)$ indicates the degree of penalty on overall dispersion of firms and the second term specifies a quadratic cost function in terms of the distance between a particular firm site and the mean site.

(III). Production Function and Utility Function

(III-1). Production Function

A firm located at $z \in \Omega \equiv [-1,1]$ is able to produce goods under the following production technology:

$$Y(z) = AK^{\alpha} L^{\beta} [Q(z)\bar{K}]^{1-\alpha-\beta} D(z)^{1-\alpha-\beta}$$

where $\alpha, \beta \in (0,1)$, $\alpha + \beta \in (0,1)$

and

D(z) is the effective land input given by

$D(z) = \min\{1, S(z)\}$ with $S(z) = s(z) \; \forall \; s(z) \geq 1$ and $S(z) = 0 \; \forall \; s(z) < 1$.

- Take output as the numéraire. Let *R(z)* denote the land rent at location *z* and let *r* and *w* denote the rental cost of capital and the wage rate, respectively. Each firm seeks to maximize its profit under the production technology specified in (1):

$$\max_{\{K,L,z\}} \pi = AK^{\alpha} L^{\beta} [Q(z)\bar{K}]^{1-\alpha-\beta} - rK - w(z)L - R(z)$$

- Given constant returns to scale in private factors, and thus zero profit in equilibrium, the conventionally defined ***bid rent function*** of firms is $R_F(z)$,

$$R_F(z) = \max_{\{K, L\}} AK^{\alpha} L^{\beta} [Q(z)\overline{K}]^{1-\alpha-\beta} - rK - w(z)L$$

- The first-order conditions with respect to K, L and location z are, respectively,

$$\alpha \frac{Y}{K} = r \tag{3}$$

$$\beta \frac{Y}{L} = w(z) \tag{4}$$

$$-2(1-\alpha-\beta)\frac{Y}{Q(z)}(z-\mu) = R_{F'}(z) + w'(z)L \tag{5}$$

(III-2). Utility Function

$\hat{U}(c,h) = U(c) \; \forall \, h \geq 1$ and $\hat{U}(c,h) = 0 \; \forall \, h < 1$

- Assume all households are identical in every respect, and each is endowed with one unit of labor and receives no disutility from work (and thus supplies one unit of labor inelastically).
- Denote by I(x,z) the net income of a household residing in x while working at z. This household earns a wage of w(z), incurs a linear commuting cost of t|x-z| and pays land rent R(x) on its one unit of land consumption.

- Given the assumptions on utility, its object is to maximize consumption c (that is equal to the net income):

$$\max_{\{x,z\}} I(x,z) = w(z) - t|x-z| - R(x)$$

- ***Locational no-arbitrage*** requires that each household must reach a constant net income for any pair of work and residential locations, $I(x,z) = I_0$, under which a household's bid rent is defined as:

$$R_C(x) = \max_z \{ w(z) - t\,|x - z|\ s.t.\ I(x,z) = I_0 \}$$

- Hence, from previous condition, under perfect competition, this means:

$$R_c(x) = w(z) - t\,|x - z| - I_0 \qquad (6)$$

for all x where consumers reside.

5.3. Equilibrium

- Under perfect competition with constant returns, each firm earns zero profit in equilibrium. Combining with (3) and (4), we have:

$$Y = \frac{1}{1 - \alpha - \beta} R_F(z)$$

- Substituting (7) into (5) to eliminate Y and applying the definition of Q yield an ordinary first-order differential equation for R_F with respect to z.

- By integration, one can express firm's bid rent as a function of the wage rate,

$$R_F(z) = \frac{2-(z-\mu)^2 - \_\sigma_\tau^2}{w(z)^{\beta/(1-\alpha-\beta)}} \Lambda_\tau(z_\tau)$$

the location index, and other exogenous parameters:

where $\Lambda_\tau(z_\tau) = R_F(z_\tau) w(z_\tau)^{\beta/(1-\alpha-\beta)} / [2-(z_\tau-\mu)^2 - \varepsilon \sigma_\tau^2]$,

depending on a reference point (z_τ) and the endogenous urban configuration (τ) to be determined.

- **Definition 1:** A ***competitive spatial equilibrium*** is a list of quantities for each location *z* {K(z), L(z), Y(z)}, prices {r(z), w(z), R(z)} and population densities {M(z), N(z)} for $z \in \Omega \equiv [-1,1]$ such that the following conditions are satisfied:

- (i) profit maximization: (3) and (4);

(ii) land rent:

$$R(z) = \text{Max}\ \{\ R_F(z)\ ,\ R_C(z)\ ,\ 1\ \}$$

$$R(z) = R_C(z) \quad if \quad N(z) > 0$$

$$R(z) = R_F(z) \quad if \quad M(z) > 0$$

$$R(-1) = R(1) = 1$$

where households' and firms' bid rent functions are given by (6) and (8);

- (iii) zero profit: (7);
- (iv) land market clearance:

$$M(z) + N(z) = 1, \quad \forall z \in \Omega$$

- (v) labor market equilibrium:

$$\int_{z \in \Omega} L(z)M(z)dz = N$$

- (vi) population balance:

$$\int_{z \in \Omega} M(z)dz = M$$

$$\int_{z \in \Omega} N(z)dz = N$$

5.4. Endogenous Determination of Urban Configuration

We determine the endogenous formation of urban configurations.

7.4.1. The Completely Mixed Urban Configuration

- A ***completely mixed*** urban configuration (denoted τ = C) is one in which firms and workers occupy every location along the linear city (see Figure 1).

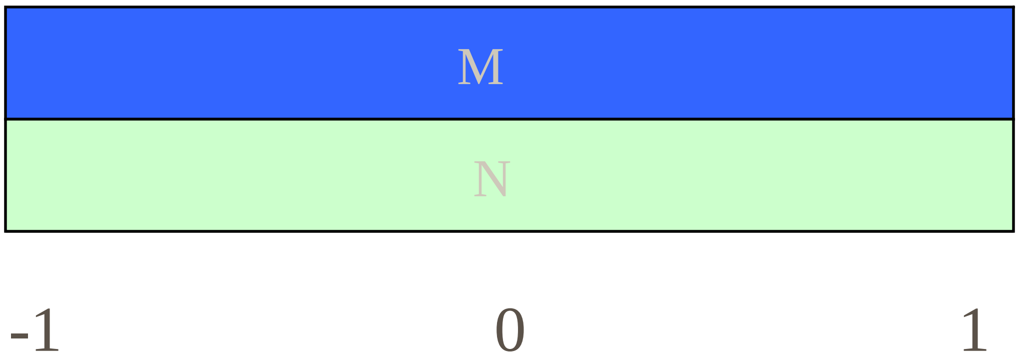

Figure 1: Completely Mixed Urban Configuration

- In this case, the mean location for firms is at the center, $\mu = 0$, and the measure of firm dispersion attains its maximum, $\sigma_C = 1$. Thus, the degree of effectiveness of interactions is:

$$Q(z) = 2 - z^2 - \varepsilon$$

- The labor market equilibrium implies $M(z)L(z) = N(z)$ for all $z \in \Omega$.

- The bid rent functions, R_F and R_C, must be identical and thus,

- Under the properties associated with the completely mixed urban configuration, consider the following condition on exogenous parameters

where and I_0

$$B_1(\varepsilon, N) = \frac{2}{1-\varepsilon}\left[1 + \frac{\beta}{1-\alpha-\beta}\frac{1}{1+I_0(\varepsilon, N)}\right]^{-1}$$

will be specified below with $\frac{\partial I_0}{\partial \varepsilon} > 0$ and $\frac{\partial I_0}{\partial N} < 0$,

Condition C:

(completely mixed urban configuration) . $B_1(\varepsilon, N) \leq t$

- This condition requires that the unit commuting cost be sufficiently large while the penalty for firm dispersion due to less effective knowledge spillovers be sufficiently low.

- **Proposition 1:**

  Under Condition C and $\frac{1-\alpha-\beta}{\beta}\frac{N}{2} < 1$,

  there is a competitive spatial equilibrium with a completely mixed symmetric urban configuration in which firms and households occupy every location in the linear city and equilibrium land rent, net income and wage (at z = 0)

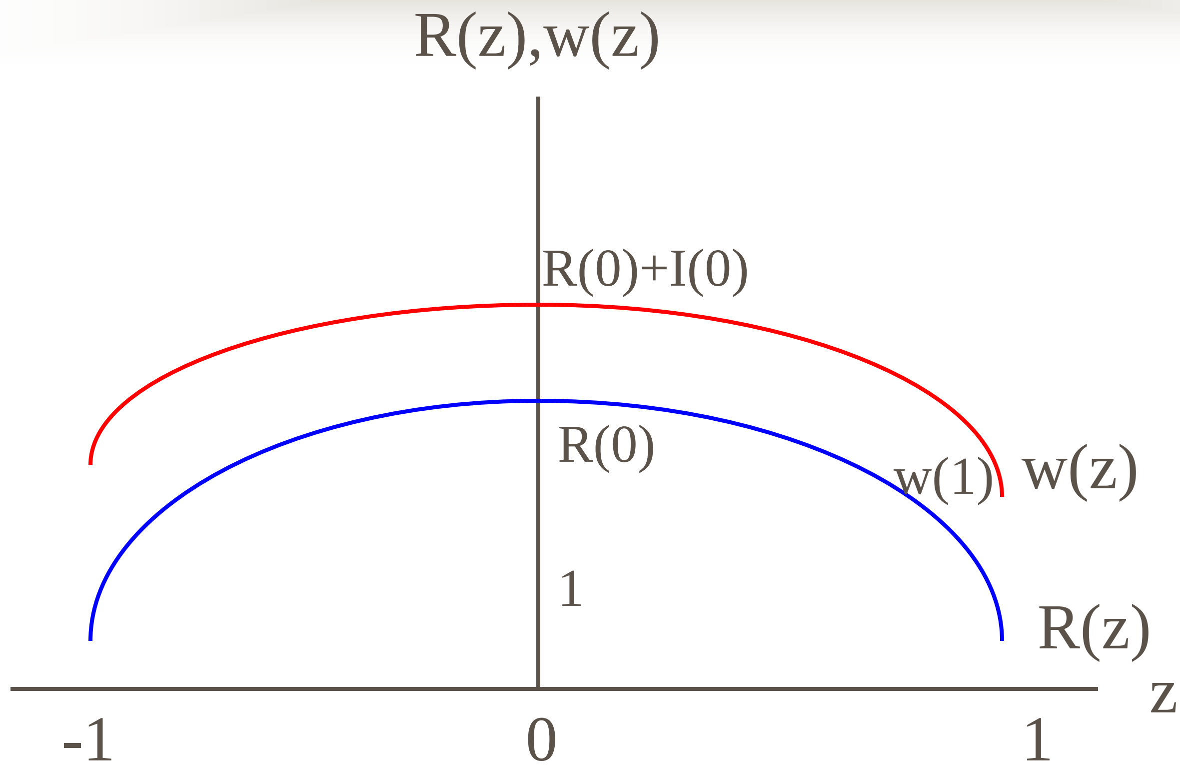

Figure 2:

Equilibrium Wage and Land rent Schedules

- Completely Mixed

- Proposition 1 indicates that this completely mixed urban configuration emerges when commuting is sufficiently costly and the penalty for firm dispersion is sufficiently low (Condition C).

- Straightforward comparative statics show that a greater penalty for firm dispersion due to less effective knowledge spillovers (a larger ε) results in an *i* increase in the equilibrium bid rent, equilibrium wage and equilibrium net income.

5.4-2 The Monocentric Urban Configuration

- ***Monocentric*** urban configuration

(denoted τ = M) in which all firms locate toward the city center within an interval [-q, q] (0 < q < 1) while households reside in the outskirts [-1, -q] and [q, 1] (see Figure 3).

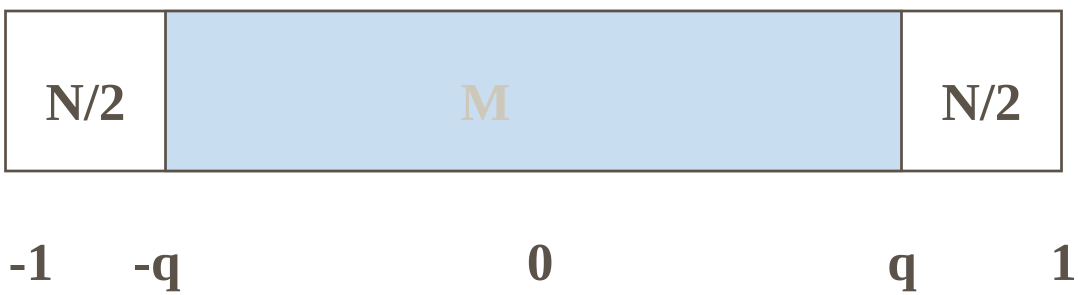

Figure 3: Monocentric Urban Configuration

- In this case, $M = 2q$, $\mu = 0$ and $\sigma_M = 2q/2 = q$,
- The degree of effectiveness of interactions:

$$Q(z) = 2 - z^2 - \varepsilon q^2 ,$$

- The firm and consumer bid rent functions must be identical:

$$R_F(q) = R_C(q)$$

Condition M: (monocentric urban configuration) .

- This condition requires that the unit commuting cost is sufficiently small while the penalty for firm dispersion is sufficiently high.

- **Proposition 2:**

Under Condition M and $\frac{2\beta}{1-\alpha} < N \le 2\left(1-\sqrt{\frac{2}{3+\varepsilon}}\right)$, *there is a competitive spatial equilibrium with a monocentric symmetric urban configuration in which all firms are clustered toward the city center [-q, q] while households reside in the outskirts of the linear city, where* , $q = 1 - \frac{1}{2}N$ *the bid rent schedule for households in the outskirts is*

$$R_C(x) = 1 + t(1 - |x|)$$

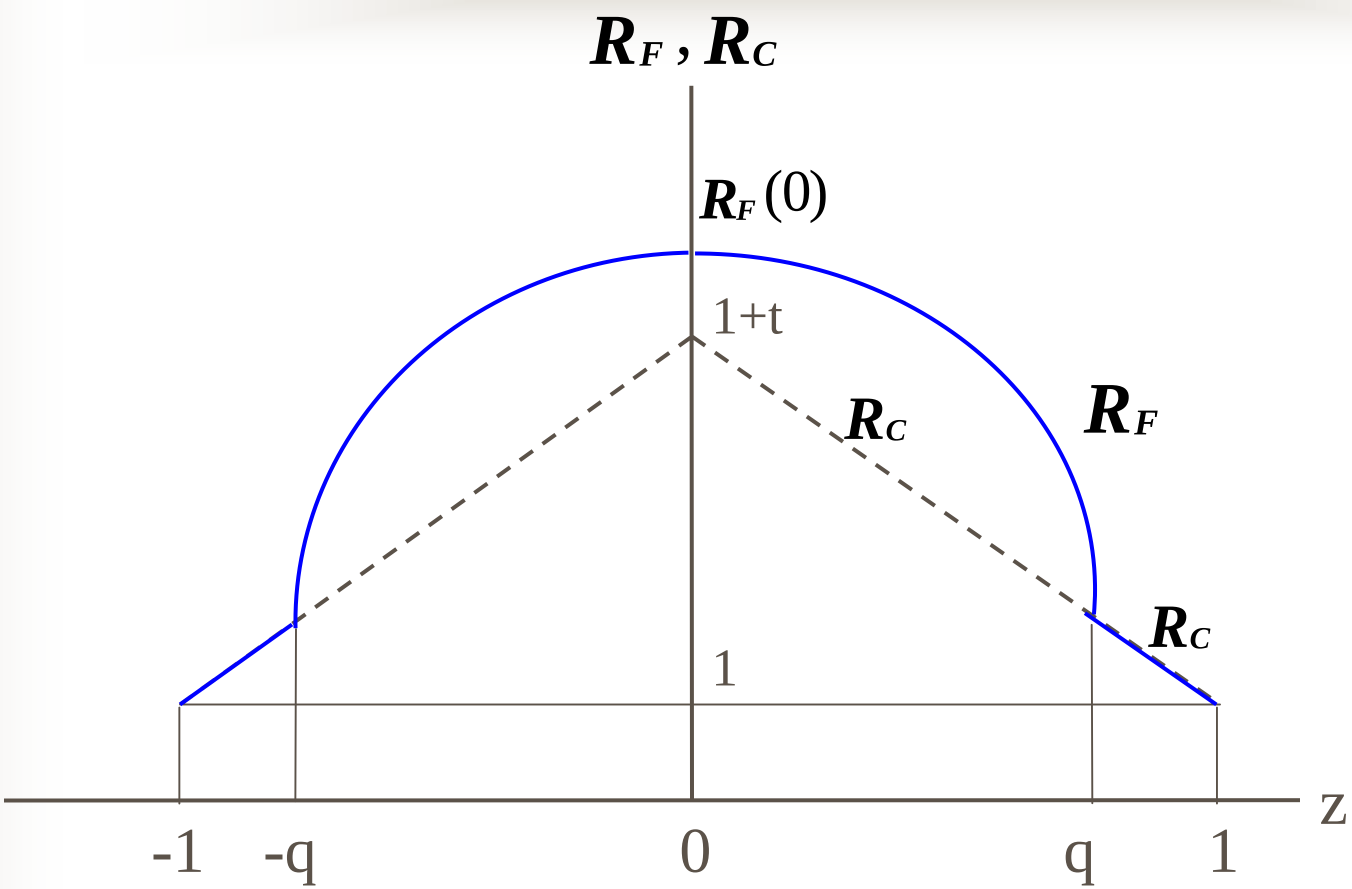

Figure 4: Equilibrium Land rent Schedules - Monocentric

- Proposition 2 suggests that when commuting cost is very low, firms are concentrated to take advantage of knowledge spillovers and households commute and receive a high wage to offset the travel costs.

- Notice that it can be shown that $B_2(\varepsilon, N) >$ $B_1(\varepsilon, N)$ for all $\varepsilon \in (0,1)$ and $N \in (0,2)$. Thus, for various values of the unit commuting cost, t, we can determine the associated equilibrium urban configuration as illustrated in Figure 5.

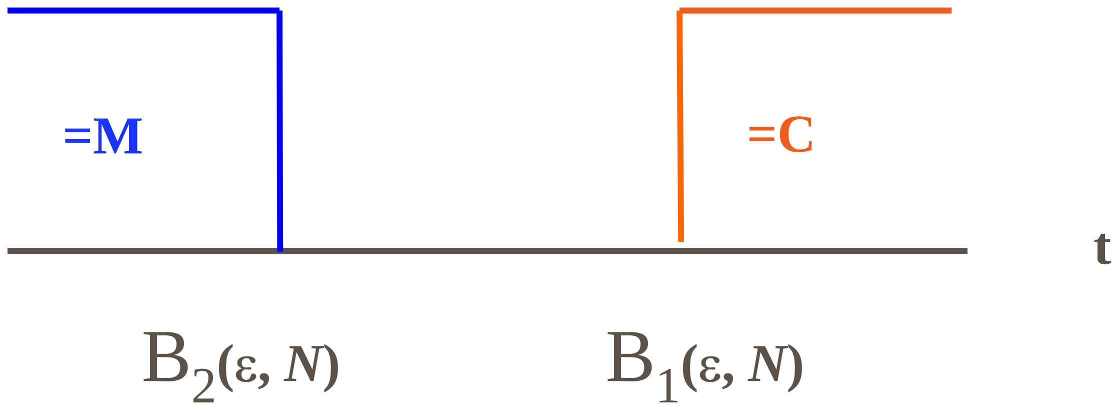

Figure 5:

Determination of Equilibrium Urban Configuration

- When the unit commuting cost is sufficiently high, the completely mixed urban configuration emerges as the unique equilibrium outcome;
- When the unit commuting cost is sufficiently low, the unique equilibrium urban configuration turns out to be monocentric.

- A question remains is what urban configuration arises in equilibrium and whether the equilibrium urban configuration is unique if the unit commuting cost is moderate [i.e., $B_2(\varepsilon, N) < t < B_1(\varepsilon, N)$] .

5.4-3 The Incompletely Mixed Urban Configuration

- The urban configuration is ***incompletely mixed*** (denoted $\tau = I$) in the sense that firms locate over the area $[-f_2, f_2]$ while households reside both toward the city center $[-f_1, f_1]$ and in the outskirts $[-1, -f_2]$ and $[f_2, 1]$, where $0 < f_1 < f_2 < 1$

(see Figure 6).

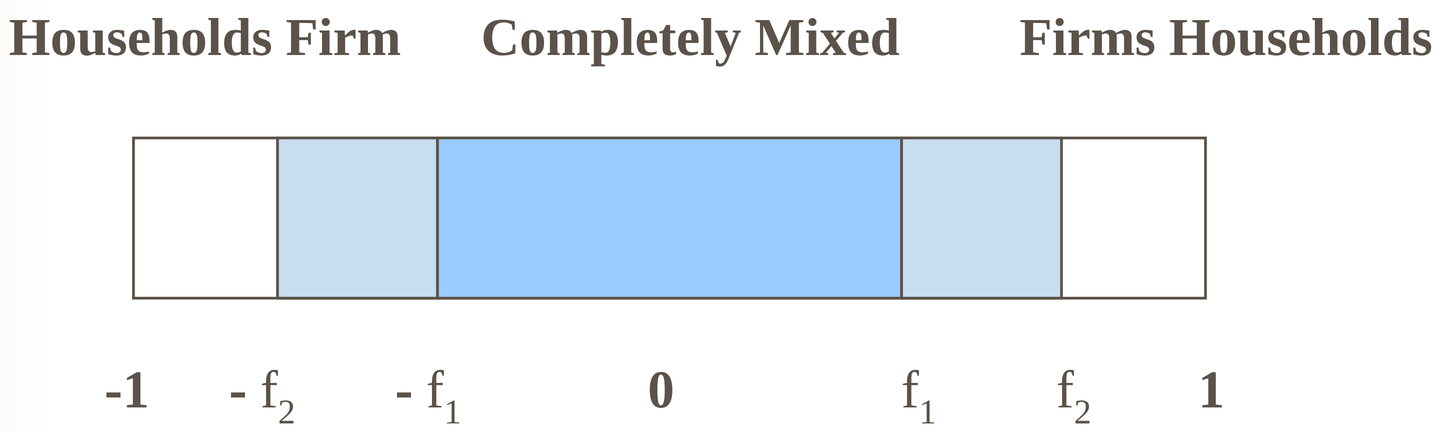

Figure 6: Incompletely Mixed Urban Configuration

- Firms and households are completely mixed only in the area around the city center. While households reside in disconnected regions, all firms locate in a connected region around the city center.

- In this case, $\mu = 0$ and $\sigma_I = 2f_2/2 = f_2$.

- The degree of effectiveness of interactions: $Q(z) = 2 - z^2 - f_2^2$.

- The equilibrium land rent function is such that

$$R(z) = R_F(z) = R_C(z) \quad \forall z \in [-f_1, f_1]$$

and $R_F(-f_2) = R_C(-f_2) = R_F(f_2) = R_C(f_2)$.

  - Consider the following condition on exogenous parameters,

$$R(z) = R_F(z) > R_C(z) \quad \forall z \in (-f_2, -f_1) \cup (f_1, f_2)$$

- **Condition I:**

(incompletely mixed urban configuration)

$$B_2(\varepsilon, N) \leq t \leq B_1(\varepsilon, N)$$

  - This condition requires a moderate unit commuting cost and a moderate penalty for firm dispersion.

- **Proposition 3:**

Under Condition I, there is a competitive spatial equilibrium with an incompletely mixed symmetric urban configuration in that firms locate over the area $[-f_2 , f_2]$ while households reside both toward the city center $[-f_1 , f_1]$ and in the outskirts $[-1, -f_2]$ and $[f_2 , 1]$ for $0 < f_1 < f_2 <1$. The land rent schedule in the outskirts $[-1, -f_2]$ and $[f_2 , 1]$ is

$$R(z) = R_C(z) = 1 + t(1 - |z|)$$

and the land rent schedule over the area $[-f_2 , f_2]$ is $R(z) = R_F(z)$

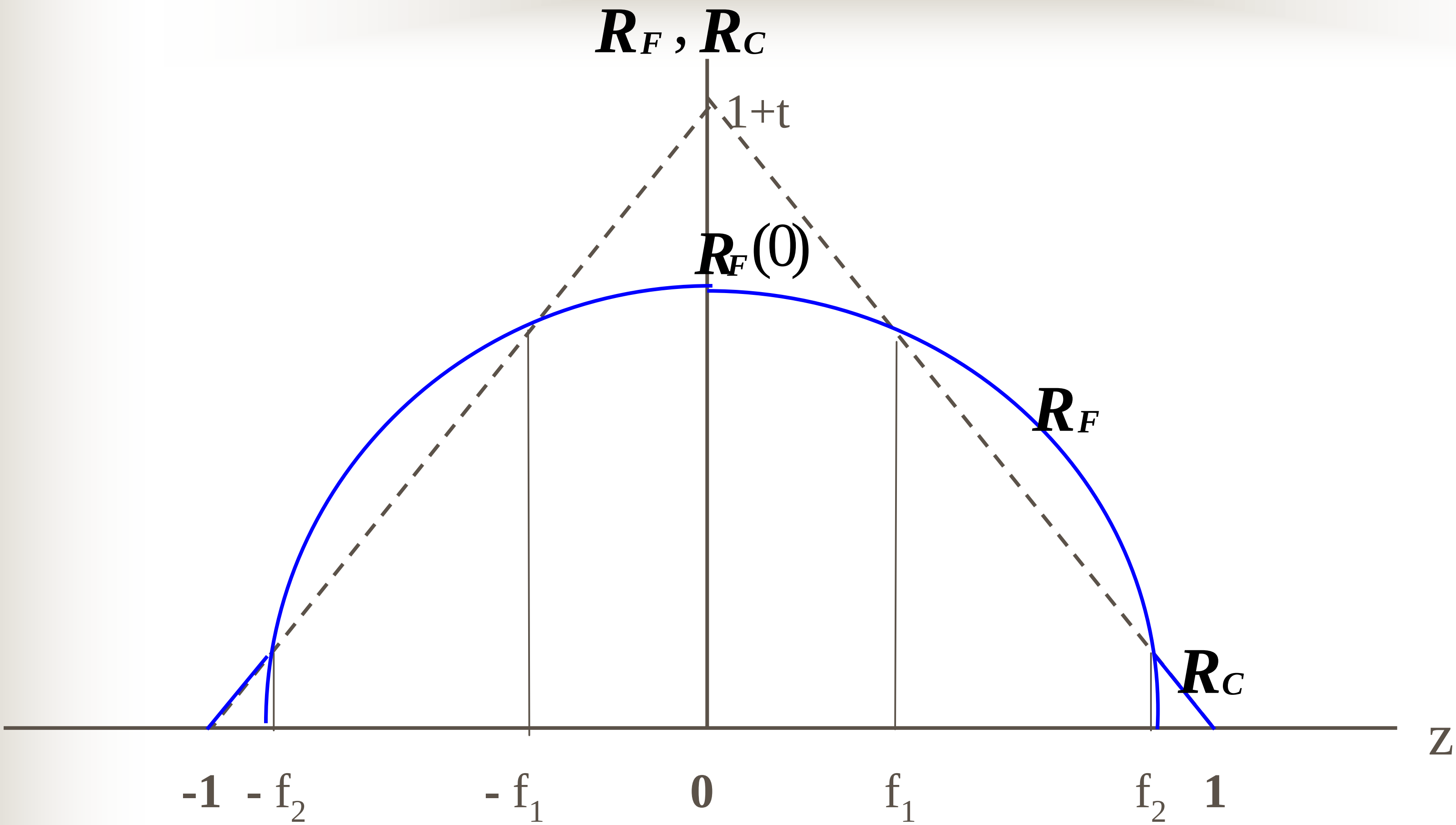

Figure 7:

Equilibrium Bid Rent Schedules- Incompletely Mixed

- This proposition states that when the unit commuting cost and the penalty for firm dispersion are moderate, the urban configuration is incompletely mixed - it is neither completely dispersed (as in the completely mixed case) nor completely concentrated (as in the monocentric case).

5.4-4 Can the Duocentric Urban Configuration be an Equilibrium Outcome?

- In the end, we would like to ask if there exists any ***multi-centric*** urban configuration, such as the ***duocentric*** city (denoted τ = D) in which firms are divided into two disconnected clusters $[-q_D, -\theta q_D]$ and $[\theta q_D, q_D]$ while households reside either around the center $[-\theta q_D, \theta q_D]$ or in the outskirts $[-1, -q_D]$ and $[q_D, 1]$, where $0 < \theta < 1$ and $0 < q_D < 1$ (see Figure 8).

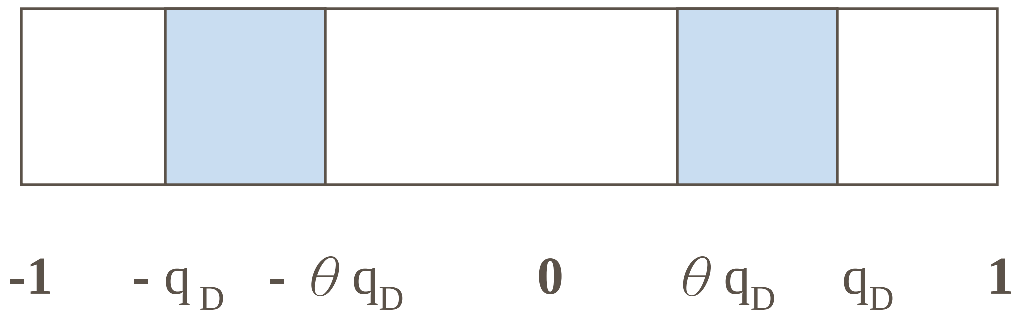

Figure 8: Duocentric Urban Configuration

- Under the duocentric urban configuration, there are two subgroups of firms - the left cluster and the right cluster. Our overall dispersion index is decomposable in firm clusters and hence still appropriate in this case.

- Straightforward calculation shows that $m(z) = M/[2(1-\theta)q_D]$ for all

$$z \in [-q_D, -\theta q_D] \cup [\theta q_D, q_D]$$

and thus $\sigma_D = (1+\theta)q_D$.

$$\boldsymbol{Q_D = \left[\ 2 - z^2 - \varepsilon\,(2v)^2 \right]}$$

- We can therefore measure the degree of effectiveness of interactions, Q_D, as:

where $v = [(1+\theta)q_D]/2$ represents the distance of the within-the-cluster mean location from the global mean location (0).

$$R(z) = R_C(z) > R_F(z) \qquad \forall z \in (-\theta q_D, \theta q_D)$$

$$R(z) = R_F(z) > R_C(z) \qquad \forall z \in (-q_D, -\theta q_D) \cup (\theta q_D, q_D)$$

$$R(z) = R_C(z) > R_F(z) \qquad \forall z \in [-1, -q_D) \cup (q_D, 1]$$

- In spatial equilibrium, we have:

and $R_F(-q_D) = R_C(-q_D) = R_F(q_D) = R_C(q_D)$ and
$R_F(-\theta q_D) = R_C(-\theta q_D) = R_F(\theta q_D) = R_C(\theta q_D)$.

- There is a crucial difference between the incompletely mixed and the duocentric urban configurations: in the latter case, the bid rent for firms in $(-\theta q_D, \theta q_D)$ is ***strictly less*** than that for households and hence the equilibrium land rent equals the household bid rent within this central cluster where no firm locates.

- **Proposition 4:** *In competitive spatial equilibrium, the duocentric symmetric urban* configuration cannot emerge.

- Importantly, under our knowledge spillover setup, a firm’s production penalty for distance from the average location of firms is strictly increasing and strictly convex, implying that it is disadvantageous for firms to be separated spatially into different clusters.

- Given such a duocentric configuration, a firm will always move to the average location of firms; the penalty, land rents, and wages are all lower there.

- As a consequence, a duocentric city in which firms are grouped into two disconnected clusters cannot be an equilibrium outcome.

- By similar arguments, any multi-centric urban configuration can be ruled out as a spatial equilibrium configuration.

5.5. Further Discussion

- In the previous section, we determine endogenously the underlying urban configuration, depending crucially on the spatial primitives of the model. Utilizing Propositions 1-3, we obtain:

- **Theorem 1:** (Existence)

  For any commuting cost and dispersion penalty parameters, there is a competitive spatial equilibrium.

- Moreover, the result in Proposition 4, in conjunction with Propositions 1-3, enables us to conclude:

- **Theorem 2:** (Uniqueness)

Almost surely in commuting cost and dispersion penalty parameters, there is a unique competitive spatial equilibrium associated with a symmetric urban configuration which is completely mixed, monocentric or incompletely mixed.

- **Proposition 5:** (Characterization of the Urban Configuration) *The almost surely unique competitive spatial equilibrium possesses the following properties:*

(i) a completely mixed symmetric urban configuration emerges when the unit commuting cost is sufficiently high, whereas a monocentric urban configuration arises when such a cost is sufficiently low;

(ii) a sufficiently large knowledge-spillover penalty on the overall dispersion of firms causes the formation of a monocentric symmetric urban configuration, but a completely mixed symmetric configuration disappears;

(iii) a sufficiently large population mass of firms induces a completely mixed symmetric urban configuration.

- The *uniqueness* of spatial equilibrium and the fact that *no* multi-centric urban configuration is an equilibrium contrast with the existing literature, such as the locational potential function framework of Fujita and Ogawa (1982), the more recent sequels using models of product differentiation [e.g., see Fujita and Krugman (1997)] and spatial competition models [e.g., see Fujita and Thisse (1986)].

- Our uniqueness property is primarily due to the strong agglomerative force from knowledge spillovers among firms. Of course, if $t = B_1(\_, N)$, the completely mixed and the incompletely mixed urban configurations can co-exist; if the $t = B_2(\_, N)$, monocentric and the incompletely mixed urban configurations can co-exist. These knife-edge cases, however, require specific combinations of some exogenous parameters, which have zero measure in the entire parameter space. Thus, our uniqueness property holds almost surely.

- This sharp contrast with Fujita and Ogawa (1982) is mainly due to the nonseparability of our Q function and the fact that the external factor is allowed to affect the factor proportion under our framework.

- More interestingly, we show that the Romer-type production externalities with a strictly convex penalty for firm dispersion and distance from the average firm location are sufficient to lead to the formation of a city in which firms are *always clustered together* in any spatial equilibrium. This eliminates the possibility of any multi-centric urban configuration, suggesting that when positive knowledge spillovers are the primary agglomerative forces, multi-centric cities cannot emerge under perfect competition.

5.6. Concluding Remarks

- Based on the Romer-type production externality, our paper has developed a general equilibrium framework under which the unique equilibrium urban configuration (completely mixed, monocentric or incompletely mixed) is determined analytically, depending on the population of firms, the commuting cost and the firms' knowledge spillover parameters.

- We show that the incorporation of distance-dependent production externalities is sufficient to ensure that firms are always clustered together in any competitive spatial equilibrium, ruling out the possibility of multi-centric urban configurations, in contrast with findings in Fujita and Ogawa (1982) and in more recent sequels employing models of product differentiation or spatial competition.

- Along these lines, there are a few straightforward extensions.

  (I) One may relax the assumptions of a fixed supply of labor and fixed demands for land.

  (II) One can investigate the usefulness of index number theory for measuring dispersedness of firms and quantifying the externality.

  (III) One may revisit our work using a discrete population model *á la* Berliant and Fujita (1992)

- The main purpose of these exercises is to check the robustness of the absence of multi-centric urban configuration and the uniqueness of competitive spatial equilibrium.

- Moreover, it may be of interest to examine the welfare properties of competitive spatial equilibrium. In particular, the presence of uncompensated knowledge spillovers may lead to a sub-optimal equilibrium - in equilibrium firms fail to account for the positive production externality and thus under-invest compared to the optimum. An intriguing question is whether such inefficiencies are lower under one urban configuration relative to others.

- Furthermore, one may add an externality to the consumer utility via local congestion or neighborhood effects. In the former case, a more dense population has a negative influence on household utility, which serves as an additional force for dispersion. In the latter case, the distance-dependent positive externality makes the incompletely mixed urban configuration (in which households are not clustered together) less likely to emerge.

- Finally, our findings provide empirically testable hypotheses regarding

  (i) the shape of the rent density,

  (ii) the locations of firms and consumers, and

  (iii) comparative statics describing the dependence of the urban configuration (measured as a discrete variable) on commuting, population and production-spillover parameters.

  - Those regarding production spillovers may also be compared with the empirical localization externality measured by Rosenthal and Strange (1998)