Lecture 4

Core-periphery Model- an Analytically Solvable Approach

1. Introduction

That make the **CP model** analytically solvable in the sense that it is possible to derive closed form solutions for the endogenous variables.

Reference:

Ottaviano, G., and R. Forslid (2003), "An analytically solvable core-periphery model," ***Journal of Economic Geography***, 3, 229-240.

- Solvability is achieved by introducing **skill heterogeneity** between workers and by coupling a **higher** level of skill with **higher** interregional mobility.

- This is in line with empirical evidence, which suggests a significantly higher geographical mobility of skilled compared to unskilled workers (Shields and Shields, ***Journal of Economic Surveys***, 3, 277-304, 1989).

2. The footloose entrepreneur model

- The economy consists of two regions, 1 and 2.
- There are two factors of production, **skilled** and **unskilled** labor.
- Each worker supplies one unit of his type of labor inelastically.

- Total endowments are ***H*** and ***L*** for **skilled** and **unskilled** labor respectively so that $H_1+H_2=\boldsymbol{H}$ and $L_1+L_2=\boldsymbol{L}$, where H_i and L_i are the endowments of the two factors in region *i*.

- **Skilled** workers can be thought of as self-employed entrepreneurs who move freely between regions, and we will therefore refer to to the model as the '***footloose entrepreneur***' (FE)model.

➡ On the demand side, preferences are defined over two final goods, a **horizontally differentiated** good X (manufactures) and a **homogenous** good A (agriculture).

$$U_i = X_i^{\mu} A_i^{1-\mu}, \qquad (1)$$

$$X_i = \left(\int_{s \in N} d_i(s)^{(\sigma-1)/\sigma} ds \right)^{\sigma/(\sigma-1)}, \qquad (2)$$

- $n_1+n_2=N$

- $\sigma > 1$ is both the **elasticity of demand** of any variety and the elasticity of substitution between any two varieties.

- Standard **utility maximization**

$$d_{ji}(s) = \frac{p_{ji}(s)^{-\sigma}}{P_i^{1-\sigma}} \mu Y_i, i, j = \{1,2\} \qquad (3)$$

$$P_i = [\int_{s \in n_i} p_{ii}(s)^{1-\sigma} ds + \int_{s \in n_j} p_{ji}(s)^{1-\sigma} ds]^{1/(1-\sigma)}, \qquad (4)$$

$$Y_i = w_i H_i + w_i^L L_i \qquad (5)$$

- The representative consumer in region i maximize utility (1) subject to the following budget constraint:

$$\int_{s\in n_i} p_{ii}(s)d_{ii}(s)ds + \int_{s\in n_j} p_{ji}(s)d_{jj}(s)ds + p_i^A A_i = Y_i, \quad (6)$$

- p^A is the price of the agricultural good.
- Turning to the supply side, firms in sector A produce a homogenous good under perfect competition and **constant returns to scale** and employ only unskilled labor.

- Without loss of generality, units are chosen so that one unit of output requires one unit of labor.
- This implies that the unit production cost for a firm in sector *A* equals the unskilled wage w_i^L .
- Then, perfect competition implies marginal cost pricing so that $p_i^A = w_i^L$.

- Firms in sector X are **monopolistically competitive** and employ both **skilled** and **unskilled** workers under increasing returns to scale.

- Product differentiation ensures a one-to-one relation between firms and varieties.

- A firm incurs a fixed input requirement of α units of skilled labor and a marginal input requirement of βx units of unskilled labor.

- The total cost of production of a firm.

$$TC_i(s) = w_i \alpha + w_i^L \beta x_i(s) \qquad (7)$$

- Given the fixed input requirement α , skilled labor market clearing implies that in equilibrium the number of firms is determined by:

$$n_i = \frac{H_i}{\alpha}, \qquad (8)$$

- The number of active firms in a region is proportional to the number of its skilled residents.

➡ **Unskilled** workers are perfectly mobile between sectors but spatially immobile and assumed to be evenly spread across regions: $L_i = L/2$.

➡ **Skilled** workers, on the contrary, are mobile and free to reside in the region that offers them the higher indirect utility (i.e., real wage).

- The assumption that skilled (geographically mobile) labor is employed in the fixed cost and unskilled (geographically immobile) labor is employed in the variable cost in the production of manufactures is the **only difference** with respect to Krugman’s CP model, where mobile labor is used in both fixed and variable cost.

- Indeed, the assumption that skilled workers are associated with the fixed cost is quite natural, since this cost often stem from headquarter services, R&D or other high-skill activities.

- However, it is enough to simplify the analysis without altering the qualitative insights of the original model.

- Analytical convenience is not the only appealing feature of the present set-up.

- Good *A* is freely traded so that its price is the same everywhere.

- Then, due to marginal cost pricing, interregional price equalization $(p_1^A = p_2^A)$ also implies interregional wage equalization $(w_1^L = w_2^L)$.

- This suggests choosing good A as numeraire, so that $p_i^A = w_i^L = 1$

.

- Trade in X, on the contrary, is inhibited by frictional trade barriers, which are modeled as **iceberg costs**: for one unit of differentiated good to reach the other region, $\tau \in [1,+\infty]$ units must be shipped.

$$\Pi_i(s) = p_{ii}(s)d_{ii}(s) + p_{ij}(s)d_{ij}(s) - \beta[d_{ii}(s) + \tau d_{ij}(s)] - \alpha w_i, \quad (9)$$

- Due to the choice of numeraire, the **unskilled** labor wage w_i^L is set equal to 1 and $\tau d_{ij}(s,t)$ represents total supply to the distant location *j* inclusive of the fraction of product that melts away in transit due to the iceberg costs.

The first order condition for maximization gives:

$$p_{ii}(s) = \beta\sigma/(\sigma-1) \quad \text{and} \quad p_{ij}(s) = \tau\beta\sigma/(\sigma-1) \quad (10)$$

➡ After using (10), the CES price index (4) simplifies to:

$$P_i = [n_i p_{ii}^{1-\sigma} + n_j p_{ji}^{1-\sigma}]^{1/(1-\sigma)}$$

$$= [n_i (\frac{\beta\sigma}{\sigma - 1})^{1-\sigma} + n_j (\frac{\tau\beta\sigma}{\sigma - 1})^{1-\sigma}]^{1/(1-\sigma)}$$

$$= (\frac{\beta\sigma}{\sigma - 1})[n_i + \tau^{1-\sigma} n_j]^{1/(1-\sigma)}$$

$$= (\frac{\beta\sigma}{\sigma - 1})[n_i + \phi n_j]^{1/(1-\sigma)}$$

Where $\phi = \tau^{1-\sigma}$

$$P_i = \frac{\beta\sigma}{\sigma - 1}[n_i + \phi n_j]^{1/(1-\sigma)}, \qquad (11)$$

➡ Where $\phi \equiv \tau^{1-\sigma} \in (0,1]$ is the ratio of total demand by domestic residents for each foreign variety to their demand for each domestic variety.

➤ It is therefore a measure of the freeness of trade, which increases τ as **falls** and is equal to one when trade is free $(\tau = 1)$.

- Since by (8) the total number of X firms is given by $n_i + n_j = H / \alpha$, the price index (11) decreases (increases) with the number of local (distant) firms.

- Due to free entry and exit, there are no profits in equilibrium.

- This implies that a firm's scale of production is such that operation profits exactly match the fixed cost paid in terms of skilled labor. (*****)

➡ That is, a firm's operation profits are entirely absorbed by the wage bill of its **skilled** workers (**??**):

$$w_i = \frac{1}{\alpha}\{p_{ii}(s)d_{ii}(s) + p_{ij}(s)d_{ij}(s) - \beta[d_{ii}(s) + \tau d_{ij}(s)]\}$$

$$w_i = \frac{\beta x_i}{\alpha(\sigma - 1)}, \qquad (12)$$

- Where $x_i = [d_{ii}(s) + \tau d_{ij}(s)]$ is total production by a typical firm in location *i*.

- This last expression can be used to determine the output of firms in regions 1 and 2.

$$x_i = d_{ii} + \tau d_{ij}$$

$$= \frac{p_{ii}^{-\sigma}}{P_i^{1-\sigma}}\mu Y_i + \tau \frac{p_{ji}^{-\sigma}}{P_j^{1-\sigma}}\mu Y_j$$

$$= \frac{(\frac{\beta\sigma}{\sigma-1})^{-\sigma}\mu Y_i}{(\frac{\beta\sigma}{\sigma-1})^{-\sigma}[n_i + \phi n_j]} + \tau \frac{(\frac{\tau\beta\sigma}{\sigma-1})^{-\sigma}\mu Y_j}{(\frac{\beta\sigma}{\sigma-1})[\phi n_i + n_j]}$$

$$= \mu(\frac{\sigma-1}{\beta\sigma})[\frac{Y_i}{[n_i + \phi n_j]} + \tau^{1-\sigma}\frac{Y_j}{[\phi n_i + n_j]}$$

$$x_i = \mu\left(\frac{\sigma - 1}{\beta\sigma}\right)\left\{\frac{Y_i}{[n_i + \phi n_j]} + \phi \frac{Y_j}{[\phi n_i + n_j]}\right\}$$

$$x_i = \frac{\sigma - 1}{\beta\sigma}\left(\frac{\mu Y_i}{n_i + \phi n_j} + \frac{\phi\mu Y_j}{\phi n_i + n_j}\right) \qquad (13)$$

➡ Using (8) and (12), equation (13) can be equivalently written as:

$$w_i = \frac{\beta x_i}{\alpha(\sigma - 1)}$$

$$= \frac{\beta}{\alpha(\sigma - 1)}\{\mu(\frac{\sigma - 1}{\beta\sigma})[\frac{Y_i}{(n_i + \phi n_j)} + \phi\frac{Y_j}{(\phi n_i + n_j)}]\}$$

$$= \frac{\mu}{\alpha}[\frac{Y_i}{(n_i + \phi n_j)} + \phi\frac{Y_j}{(\phi n_i + n_j)}]$$

$$w_i = \frac{\mu}{\sigma}[\frac{Y_i}{(H_i + \phi H_j)} + \frac{\phi Y_j}{(\phi H_i + H_j)}] \qquad (14)$$

- Where, by (5) and (8), income equals:

$$Y_i = \frac{L}{2} + w_i H_{i.} \qquad (15)$$

$$n_1 = \frac{H_1}{\alpha}$$

$$n_2 = \frac{H_2}{\alpha}$$

$$P_1 = \frac{\beta\sigma}{\sigma - 1}[n_1 + \phi n_2]^{\frac{1}{1-\sigma}}$$

$$P_2 = \frac{\beta\sigma}{\sigma - 1}[\phi n_1 + n_2]^{\frac{1}{1-\sigma}}$$

$$w_1 = \frac{\beta x_1}{\alpha(\sigma - 1)}$$

$$w_2 = \frac{\beta x_2}{\alpha(\sigma - 1)}$$

$$x_1 = \mu(\frac{\sigma - 1}{\beta\sigma})(\frac{Y_1}{n_1 + \phi n_2} + \frac{\phi Y_2}{\phi n_1 + n_2})$$

$$x_2 = \mu(\frac{\sigma - 1}{\beta\sigma})(\frac{\phi Y_1}{\phi n_1 + n_2} + \frac{Y_2}{n_1 + \phi n_2})$$

$$Y_1 = \frac{1}{2}L + w_1 H_1$$

$$Y_2 = \frac{1}{2}L + w_2 H_2$$

$$H_1 + H_2 = H$$

$$\frac{w_1}{P_1^{\mu}} = \frac{w_2}{P_2^{\mu}}$$

➡ The system of 12 equations simultaneously determined 11 endogenous variables

$$\{n_1, n_2, P_1, P_2, w_1, w_2, x_1, x_2, Y_1, Y_2, H_1, H_2\}$$

Homework: Try to solve the equation (16) in the page 233 of the paper

$$w_i = \frac{(\mu/\sigma)}{1-(\mu/\sigma)} \frac{L}{2} \frac{2\phi H_i + [1-(\mu/\sigma)+(1+(\mu/\sigma))\phi^2]H_j}{\phi(H_i^2 + H_j^2) + [1-(\mu/\sigma)+(1+(\mu/\sigma))\phi^2]H_i H_j} \qquad (16)$$

➡ Defining $h = H_1/H$ as the share of **skilled** workers that reside in region 1,

$$\frac{w_1}{w_2} = \frac{2\phi h + [1-(\mu/\sigma)+(1+(\mu/\sigma))\phi^2](1-h)}{2\phi(1-h) + [1-(\mu/\sigma)+(1+(\mu/\sigma))\phi^2]h} \qquad (17)$$

➡ Differentiation (17) with respect to h shows that the region with more skilled workers offers a higher (lower) skilled worker wage whenever ϕ is larger (smaller) than the threshold:

$$w = \frac{w_1}{w_2} = \frac{2\phi h + [1 - (\mu/\sigma) + (1 + (\mu/\sigma))\phi^2](1-h)}{2\phi(1-h) + [1 - (\mu/\sigma) + (1 + (\mu/\sigma))\phi^2]h}$$

$$\frac{dw}{dh}=\frac{1}{\{2\phi(1-h)+[1-(\mu/\sigma)+(1+(\mu/\sigma))\phi^2]h\}^2}\cdot$$

$$\{2\phi(1-h)+[1-(\mu/\sigma)+(1+(\mu/\sigma))\phi^2]h\}\cdot\ \{2\phi-[1-(\mu/\sigma)+(1+(\mu/\sigma))\phi^2]\}$$

$$-\{2\phi h+[1-(\mu/\sigma)+(1+(\mu/\sigma)](1-h)\}\ \{(1-(\mu/\sigma)+(1+(\mu/\sigma))\phi^2]-2\phi\}$$

$$=0$$

Set $\Theta \equiv [1 - (\mu/\sigma) + (1 + (\mu/\sigma))\phi^2] > 0,$

$$[2\phi(1-h) + \Theta h][2\phi - \Theta] - [2\phi h + \Theta(1-h)][\Theta - 2\phi] = 0$$

$$[2\phi(1-h) + \Theta h][2\phi - \Theta] + [2\phi h + \Theta(1-h)][2\phi - \Theta] = 0$$

$$[2\phi - \Theta]\{2\phi(1-h) + \Theta h + 2\phi h + \Theta(1-h)\} = 0$$

$$[2\phi - \Theta][2\phi + \Theta] = 0$$

$$2\phi - \Theta = 0$$

Con't

$$1-\frac{\mu}{\sigma}+[1+(\mu/\sigma)]\phi^2=2\phi$$

$$[1+(\mu/\sigma)]\phi^2-2\phi+[1-(\mu/\sigma)]=0$$

$$\{[1+(\mu/\sigma)]\phi-[1-(\mu/\sigma)]\}(\phi-1)=0$$

- Since $\phi\equiv\tau^{1-\sigma}, \phi=1$ implies $\tau=1$,

And $\tau=1$ represents non transportation (or trade) cost.

- Free Trade.

Thus $$(1+\frac{\mu}{\sigma})\phi = (1-\frac{\mu}{\sigma})$$

$$\phi_w = \frac{1-(\mu/\sigma)}{1+(\mu/\sigma)}$$

[equation (18) in page 234]

$$\phi_w = \frac{1-(\mu/\sigma)}{1+(\mu/\sigma)} \qquad (18)$$

With $\phi_w \in (0,1)$. This is the result of a trade-off between two opposing forces.

- On the one hand, given trade costs, a larger number of skilled workers in a certain region entail a larger number of competing manufacturing firms.

- For given expenditures on manufactures, that depressed the local price index inducing a fall in local demand per firm (‘market crowding’ effect).

- Lower demand leads to lower operating profits and, therefore, lower skilled wages.

- On the other hand, hosting more firms also implies additional operation profits and thus additional skilled income, a fraction of which is spent on local manufactures.

➡ Accordingly, local expenditures are larger, which, for a given price index, increases demand per firm ('market size' effect).

➤When $\phi = \phi_w$ the two effects exactly offset each other, while the former (latter) dominates the latter (former) whenever ϕ is smaller (larger) than ϕ_w .

➡ Notice that this is a 'global' analytical result (i.e. true for every *h*) which is not available in the CP model.

3. Equilibrium and Stability

$$\dot{h} \equiv dh/dt = \begin{cases} W(h,\phi) & \text{if } 0 < h < 1 \\ \min\{0, W(h,\phi)\} & \text{if } h = 1 \\ \max\{0, W(h,\phi)\} & \text{if } h = 0 \end{cases} \qquad (19)$$

$W(h,\phi)$ is the current indirect utility differential associated with (1):

$$W(h,\phi) \equiv \eta\left(\frac{w_1}{P_1^{\mu}} - \frac{w_2}{P_2^{\mu}}\right) \qquad (20)$$

Where $\eta \equiv \mu^{\mu}(1-\mu)^{1-\mu}$. Moreover, by (8) and (11), the two price indexes are:

$$P_1 = \frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{1/(1-\sigma)}[h+\phi(1-h)]^{\frac{1}{1-\sigma}} \qquad (21)$$

$$P_2 = \frac{\beta\sigma}{\sigma-1}\left(\frac{H}{\alpha}\right)^{1/(1-\sigma)}[1-h+\phi h]^{\frac{1}{1-\sigma}} \qquad (22)$$

- The presence of P_1 and P_2 in (20) adds a new item to the list of location effects.

- In particular, (21) and (22) show that, for a given wage, the region with more **skilled workers**, and thus more manufacturing firms, grants higher purchasing power, that is, higher consumer surplus.

- The reason is its **lower price index** as the larger number of domestic firms implies that fewer manufacturing varieties are imported and burdened by trade costs (‘cost-of-living’ effect).

- Therefore, this additional effect teams up with the market size effect to support the agglomeration of manufactures against the opposition of the market crowding effect.

Substituting (21) and (22) in (20) we obtain:

$$W(h,\phi)=\frac{\Phi}{\phi(h^2+(1-h)^2+(1-(\mu/\sigma)+(1+(\mu/\sigma))\phi^2)h(1-h)}\cdot V(h,\phi), \quad (23)$$

where $\Phi\equiv[\eta\mu L\alpha^{\mu/(1-\sigma)}(\sigma-1)^{\mu}]/[2(\sigma-\mu)H^{(1+\mu-\sigma)/(1-\sigma)}(\sigma\beta)^{\mu}]>0,$

$$V(h,\phi) \equiv \frac{2\phi h + [1-(\mu/\sigma)+(1+(\mu/\sigma)\phi^2](1-h)}{[h+\phi(1-h)]^{\mu/(1-\sigma)}}$$

$$-\frac{2\phi(1-h)+[1-(\mu/\sigma)+(1+(\mu/\sigma)\phi^2]h}{[(1-h)+\phi h]^{\mu/(1-\sigma)}} \qquad (24)$$

A corner configuration $(h = 0 \text{ or } h = 1)$ is always stable when it is an equilibrium, while an interior equilibrium

$(0 < h < 1)$ is stable if and only if the slope of $W(h, \phi)$ is non-positive in its neighborhood.

- Inspection of (23) reveals that for the determination of equilibria all that matters is $V(h,\phi)$. In particular, all interior equilibria are solutions to $V(h,\phi)=0$

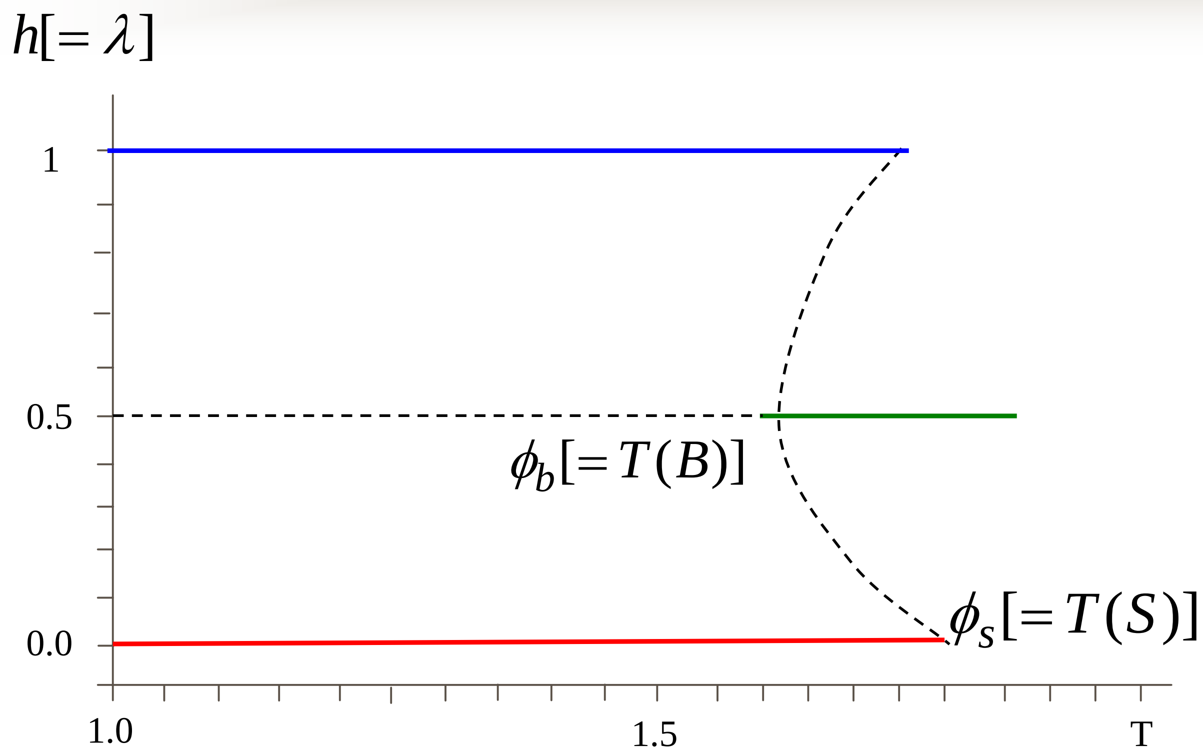
- while fully agglomerated configurations

$h=0$ or $h=1$ are equilibria if and only if $V(1,\phi)>0$ and $V(0,\phi)<0$.

$$V(0,\phi) = -V(1,\phi) = \frac{[1-(\mu/\sigma)+(1+(\mu/\sigma))\phi^2]}{\phi^{\mu/(1-\sigma)}} - 2\phi$$

$$1-\frac{\mu}{\sigma}+(1+\frac{\mu}{\sigma})(\phi_s)^2 - 2(\phi_s)^{1+\mu/(1-\sigma)} = 0 \qquad (25)$$

Where ϕ_s is what Fujita et al. (1999) call the 'sustain point' T(S) (Chapter 5, page 68).



Core-periphery bifurcation (P.68, Ch.5)

➡Turning to interior equilibria we can prove that

$V(h,\phi)=0$ has at most three solutions for $0<h<1$. It is readily verified that one solution exists for any values of parameters.

➤This is the symmetric outcome $h=1/2$, which entails an even geographical distribution of skilled workers and modern firms.

- This solution is stable whenever $V_h(1/2,\phi) < 0$.

- This is the case if and only if trade costs are so large that ϕ is below the threshold value ϕ_b defined as:

$$V(\frac{1}{2},\phi)=\frac{1}{[\frac{1}{2}+\frac{1}{2}\phi]^{\mu/(1-\sigma)}}\{\phi+[1-\frac{\mu}{\sigma}+(1+\frac{\mu}{\sigma})\phi^2]\frac{1}{2}-\phi-\frac{1}{2}[1-\frac{\mu}{\sigma}+(1+\frac{\mu}{\sigma})\phi^2]\}$$

$$=0,\Rightarrow \phi_b$$

- ϕ_b is what Fujita et al. (1999) call the 'break point' T(B). (Chapter 5, page 68)

$$\frac{\partial V(h,\phi)}{\partial h}$$

$$= \frac{1}{\{[h+\phi(1-h)]^{\mu/(1-\sigma)}\}^2}\{[h+\phi(1-h)]^{\frac{\mu}{1-\sigma}}(2\phi-\Phi)-$$

$$[2\phi h+\Phi(1-h)]\frac{\mu}{1-\sigma}[h+\phi(1-h)]^{\frac{\mu}{1-\sigma}-1}(1-\phi)\}$$

$$-\frac{1}{\{[(1-h)+\phi h]^{\mu/(1-\sigma)}\}^2}\{[(1-h)+\phi h]^{\frac{\mu}{1-\sigma}}(-2\phi+\Phi)-$$

$$[2\phi(1-h)+\Phi h]\frac{\mu}{1-\sigma}[(1-h)+\phi h]^{\frac{\mu}{1-\sigma}-1}(1-\phi)\}$$

$$= \frac{1}{\{[h+\phi(1-h)]^{\mu/(1-\sigma)}\}^2}\{[h+\phi(1-h)]^{\frac{\mu}{1-\sigma}}(2\phi-\Phi)-$$

$$\frac{\mu(1-\phi)}{1-\sigma}[2\phi h+\Phi(1-h)][h+\phi(1-h)]^{\frac{\mu}{1-\sigma}-1}\}$$

$$-\frac{1}{\{[(1-h)+\phi h]^{\mu/(1-\sigma)}\}^2}\{[(1-h)+\phi h]^{\frac{\mu}{1-\sigma}}(\Phi-2\phi)+$$

$$\frac{\mu(1+\phi)}{1-\sigma}[2\phi(1-h)+\Phi h][(1-h)+\phi h]^{\frac{\mu}{1-\sigma}-1}\}$$

Con’t

$$\left.\frac{\partial V(h,\phi)}{\partial h}\right|_{h=\frac{1}{2}} \le 0$$

$$2(2\phi-\Phi)(\frac{1}{2}+\frac{1}{2}\phi)^{\frac{\mu}{1-\sigma}}-2\frac{\mu(1+\phi)}{1-\sigma}(\phi+\frac{1}{2}\Phi)(\frac{1}{2}+\frac{1}{2}\phi]^{\frac{\mu}{1-\sigma}-1}\le 0$$

$$2(\frac{1}{2}+\frac{1}{2}\phi)^{\frac{\mu}{1-\sigma}}[(2\phi-\Phi)-\frac{\mu(1+\phi)}{1-\sigma}(\phi+\frac{1}{2}\Phi)(\frac{1}{2}+\frac{1}{2}\phi)^{-1}]\le 0$$

$$2\phi - \Phi - \frac{\mu(1+\phi)}{1-\sigma} \frac{\phi + \frac{1}{2}\Phi}{\frac{1}{2} + \frac{1}{2}\phi} \leq 0$$

$$2\phi - \Phi \leq \frac{\mu(1+\phi)}{1-\sigma} \frac{\phi + \frac{1}{2}\Phi}{\frac{1}{2} + \frac{1}{2}\phi}$$

$$2\phi - 1 + \frac{\mu}{\sigma} - (1 + \frac{\mu}{\sigma})\phi^2 \leq \frac{\mu(1+\phi)}{1-\sigma} \frac{\phi + \frac{1}{2} - \frac{1}{2}\frac{\mu}{\sigma} + \frac{1}{2}(1 + \frac{\mu}{\sigma})\phi^2}{\frac{1}{2} + \frac{1}{2}\phi}$$

$$\phi_b \equiv \phi_w \frac{(1 - 1/\sigma - \mu/\sigma)}{(1 - 1/\sigma + \mu/\sigma)}, \qquad (26)$$

Moreover, if $\phi_b < 0$ the symmetric outcome is never stable independently of parameter values:

- the market crowding effect is always dominated by market size and cost-of-living effects.
- We rule out this case by assuming that $\mu < \sigma - 1$ (the 'no-black-hole' condition).

- Also that the cost-of-living effect always works in favor of the large country.

- At the breakpoint, where real wages are equal, the wage effect must go in favor of the small location so that

$$\phi_b < \phi_w.$$

- Apart from $h = 1/2$, there exist at most two other interior equilibria that are symmetrically placed around it.

- Which is symmetric around $h = 1/2$ and changes concavity at most twice.

$$W(1/2,\phi) = 0 \forall \phi \qquad (27)$$

$$W_h(1/2,\phi_b) = 0, W_{h\phi}(1/2,\phi_b) > 0 \qquad (28)$$

$$W_{hh}(1/2,\phi_b) = 0, W_{hhh}(1/2,\phi_b) > 0 \qquad (29)$$

- Property (27) says that $h = 1/2$ is always a steady state (persistent steady state).

- Properties (28) say that as ϕ increases from 0, the steady state $h = 1/2$ turns from stable to unstable as soon as rises above ϕ_b .

- Properties (29) say that, as soon as the steady state $h = 1/2$ changes stability, two additional steady states appear in its neighborhood.

➡ All these properties together say that the differential equation (19) undergoes a (local) ‘pitchfork bifurcation’ at $.\phi = \phi_b$

➡ Since $W_{hhh}(1/2, \phi_b) > 0$ the bifurcation is ‘subcritical :

as ϕ falls ϕ_b below the persistent steady state $h = 1/2$ gains stability giving rise to two unstable symmetric steady states in its neighborhood (Guckenheimer and Holmes, 1990).

➡ The difference here is that, given the explicit function , one can claim that they hold also globally.

➤ In other words, while in Krugman's model analytical results are only local and numerical methods are required to investigate its global properties, here both local and global properties can be assessed analytically.