



Lecture 2

Monopolistic Competition and Optimum

Product Differentiation

Reference:

Dixit and Stiglitz, 1977, Monopolistic Competition and Optimum Product Diversity, *American Economic Review* 67, 297-308.

2.1 Pricing Behavior

The utility function for consumers

$$u = u(x_0, v(x_1, x_2, \dots, x_n)) \quad (2.1)$$



Assumption:

- (1) A **separable utility function** with convex indifference surfaces.
- (2) V is a **symmetric function**, thus all commodities have equal fixed and marginal costs.
- (3) Commodities with a pair close together have better mutual substitutes than a pair farther apart.
- (4) u can be regarded as representing Samuelsonian social indifference curves, or regarded as a multiple of a **representative consumer's utility**.
- (5) Product diversity can be interpreted either as different consumers using different varieties, or as diversification on the part of each consumer.



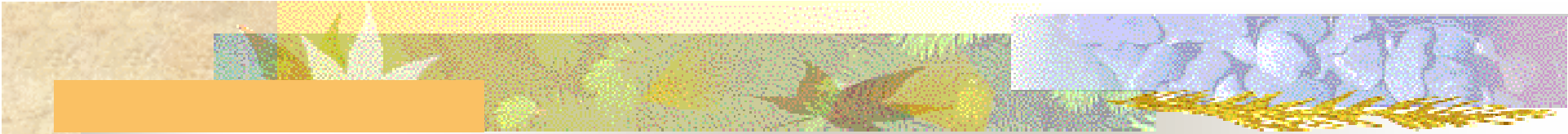
◎ The Characteristics of Monopolistic Competition

- (1) The **number of firms** is large enough.
- (2) The commodities produced by each firm are good substitutes among themselves.
- (3) Firms enter the market until the next potential entrant would make a loss.
- (4) Each firm has not the dominant power to determine the commodity's price, it is also not as perfect competition market to take the price as given variable (i.e., exogenously determined)

◎Utility Function:

$$u = u(x_0, [\sum_{i=1}^n x_i^\rho]^\frac{1}{\rho}) \equiv x_0 + (\sum_{i=1}^n x_i^\rho)^\frac{1}{\rho} \quad (2.2)$$

- (i) For **concavity**, we need $\rho < 1$
- (ii) For allowing a situation where several of the x_i are zero, we need $\rho > 0$
- (iii) ρ denotes **the substitution index** among commodities, $\rho \in (0,1)$, it also interprets **the preference of diversity commodities** for the consumers, the smaller of the ρ , implies the consumers prefer to consume the commodities more diversity.
- (iv) u is a **homothetic function** in its arguments.


$$\mathit{Max}_{x_0, x_i} U(x_0, [\sum_{i=1}^n x_i^\rho]^\frac{1}{\rho}) \quad (2.3)$$

$$\text{s.t} \quad x_0 + \sum_{i=1}^n p_i x_i = I \quad (2.4)$$

Lagrange function:

$$L = x_0 + (\sum_{i=1}^n x_i^\rho)^\frac{1}{\rho} + \lambda(I - \sum_{i=1}^n p_i x_i - x_0) \quad (2.5)$$

λ : Lagrange parameter, it denotes the **marginal contribution of income to utility.**

◎ Budget Constraint:

$$x_0 + \sum_{i=1}^n p_i x_i = I \quad (2.6)$$

$$\frac{\partial L}{\partial x_0} = 1 - \lambda = 0$$

$$\lambda = 1$$

$$\frac{\partial L}{\partial x_i} = \frac{1}{\rho} \left(\sum_{i=1}^n x_i^\rho \right)^{\frac{1}{\rho}-1} \cdot \rho x_i^{\rho-1} - \lambda p_i = 0$$

$$\Rightarrow \left(\sum_{i=1}^n x_i^\rho \right)^{\frac{1}{\rho}-1} x_i^{\rho-1} = \lambda p_i$$

$$\Rightarrow \left(\sum_{i=1}^n x_i^\rho \right)^{\frac{1-\rho}{\rho}} x_i^{\rho-1} = \lambda p_i$$

$$\Rightarrow \left[\left(\sum_{i=1}^n x_i^\rho \right)^{\frac{1}{\rho}} \right]^{1-\rho} x_i^{\rho-1} = \lambda p_i \quad (2.7)$$

$$\Rightarrow \left[\left(\sum_{i=1}^n x_i^\rho \right)^{\frac{1}{\rho}} \right]^{1-\rho} x_i^{\rho-1} = p_i \quad (2.8)$$

Therefore, we have:

$$(V)^{1-\rho} x_i^{\rho-1} = p_i \quad (2.8')$$

In turn:

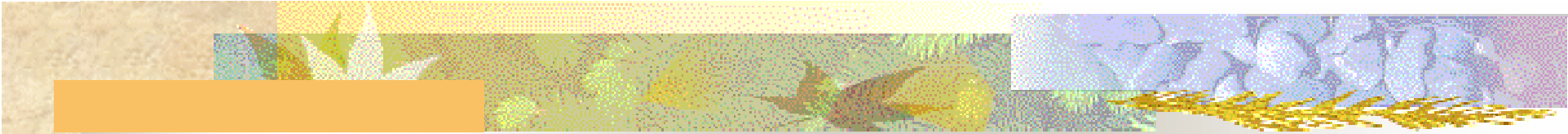
$$x_i = (p_i)^{\frac{1}{\rho-1}} V \quad (2.9)$$

Where: $V \equiv \left(\sum_{i=1}^n x_i^{\rho} \right)^{\frac{1}{\rho}}$

$$\begin{aligned} \frac{\partial x_i}{\partial p_i} &= \frac{1}{\rho-1} p_i^{\frac{1}{\rho-1}-1} V \\ &= \frac{1}{\rho-1} \frac{p_i^{\frac{1}{\rho-1}}}{p_i} V \\ &= \frac{1}{\rho-1} \frac{x_i}{p_i} \end{aligned} \quad (2.10)$$

$$\begin{aligned}
 \varepsilon &= -\frac{\frac{\partial x_i}{x_i}}{\frac{\partial p_i}{p_i}} = -\frac{\partial x_i}{\partial p_i} \frac{p_i}{x_i} \\
 &= -\frac{1}{\rho - 1} \frac{x_i}{p_i} \frac{p_i}{x_i} \\
 &= \frac{1}{1 - \rho} \qquad (2.11)
 \end{aligned}$$

→ the **higher** ρ , the **larger** ε , it implies the **close substitution** among commodities, that is, the difference between a pair of commodities become **smaller**.



◎ If we assume there are **symmetric** across the commodities x_i , that is, the consumers buy each commodity with the **same amount**, then the utility function can be simplified as:

$$V = \left(\sum_{i=1}^n x_i^\rho \right)^{\frac{1}{\rho}}$$

$$= (nx^\rho)^{\frac{1}{\rho}}$$

$$= n^{\frac{1}{\rho}} x \quad (7.12)$$

$$\Rightarrow V_e = \ln V = \frac{1}{\rho} \ln n + \ln x$$

$$V = e^{V_e} = e^{\frac{1}{\rho} \ln n + \ln x} \quad (2.13)$$

$$\begin{aligned}
 \text{(i)} \quad \frac{\partial V}{\partial \rho} &= e^{Ve} \cdot \frac{\partial V_l}{\partial \rho} = e^{\ln n^{\frac{1}{\rho}} x} \cdot \left(-\frac{1}{\rho^2}\right) \ln \cdot n \\
 &= -\frac{1}{e^2} n^{\frac{1}{\rho}} \cdot x \cdot \ln n < 0
 \end{aligned} \tag{2.14}$$

$$\text{(ii)} \quad \frac{\partial V}{\partial n} = \frac{1}{\rho} n^{\frac{1}{\rho}-1} \cdot x = \frac{1}{\rho} n^{\frac{1-\rho}{\rho}} \cdot x > 0 \tag{2.15}$$

➔ Given n , the **smaller** ρ implies consumers prefer to consume commodities **more diversity**, and thus the **smaller** ρ among commodities, the **higher** utility level for the consumers enjoying.

➔ Given ρ the **large number** of n , the **higher** the utility level can achieved.

Firm's Profit function:

$$\pi(x_i) = p_i x_i - c x_i - F \quad (2.16)$$

$$\begin{aligned} \max_{x_i} \pi(x_i) \quad \frac{\partial \pi(x_i)}{\partial x_i} &= p_i + x_i \frac{\partial p_i}{\partial x_i} - c \\ &= p_i + x_i \frac{1}{\frac{\partial x_i}{\partial p_i}} - c \\ &= p_i + x_i \frac{1}{\frac{1}{\rho - 1} \frac{x_i}{p_i}} - c \\ &= p_i + (\rho - 1) p_i - c = 0 \end{aligned}$$

$$\rho p_i = c$$

$$p_i^* = \frac{1}{\rho} c \quad (2.17)$$

S.O.C

$$\frac{\partial^2 \pi}{\partial x_i^2} = \frac{\partial p_i}{\partial x_i} + \frac{\partial p_i}{\partial x_i} + x_i \frac{\partial^2 p_i}{\partial x_i^2}$$

$$= 2 \frac{\partial p_i}{\partial x_i} + x_i \frac{\partial^2 p_i}{\partial x_i^2}$$

$$= 2 \frac{1}{\frac{\partial x_i}{\partial p_i}} + x_i \frac{\partial^2 p_i}{\partial x_i^2}$$

$$= 2 \frac{1}{\frac{1}{\rho-1} \frac{x_i}{p_i}} + x_i \frac{\partial^2 p_i}{\partial x_i^2}$$

$$= 2 \frac{(\rho-1)p_i}{x_i} + x_i \frac{\rho-1}{x_i} \left[-\frac{p_i}{x_i} + \frac{\partial p_i}{\partial x_i} \right]$$

$$= 2 \frac{(\rho - 1)p_i}{x_i} + (\rho - 1) \left[-\frac{p_i}{x_i} + (\rho - 1) \frac{p_i}{x_i} \right]$$

$$= 2 \frac{(\rho - 1)p_i}{x_i} + (\rho - 1) \frac{p_i}{x_i} (\rho - 2)$$

$$= (\rho - 1) \frac{p_i}{x_i} [2 + \rho - 2]$$

$$= \rho(\rho - 1) \frac{p_i}{x_i} < 0, \quad (2.18)$$

where

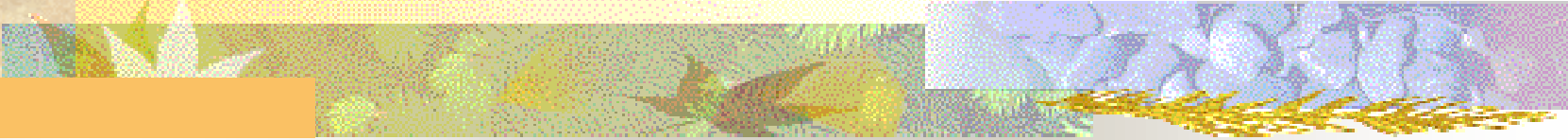
$$\frac{\partial p_i}{\partial x_i} = (\rho - 1) \frac{p_i}{x_i} \quad (2.19)$$

$$\frac{\partial^2 p_i}{\partial x_i^2} = -(\rho - 1) \frac{p_i}{x_i^2} + (\rho - 1) \frac{1}{x_i} \frac{\partial p_i}{\partial x_i} \quad (2.20)$$

$$\max_{p_i} \pi_i \quad \frac{\partial \pi_i}{\partial p_i} = x_i + p_i \frac{\partial x_i}{\partial p_i} - c \frac{\partial x_i}{\partial p_i} = 0$$

$$\Rightarrow x_i + (p - c) \left(\frac{\partial x_i}{\partial p_i} \right) = 0$$

$$\begin{aligned} x_i &= (p - c) \left(-\frac{\partial x_i}{\partial p_i} \right) \\ &= (p - c) \left(-\frac{1}{\rho - 1} \frac{x_i}{p_i} \right) \\ &= (p - c) \left(\frac{1}{1 - \rho} \right) \frac{x_i}{p_i} \end{aligned} \quad (2.21)$$


$$1 = \left(\frac{1}{1-\rho}\right) \frac{p_i - c}{p_i}$$


$$(1-\rho) = \frac{p_i - c}{p_i}$$

$$(1-\rho)p_i = p_i - c$$

$$c = [1 - (1-\rho)]p_i$$

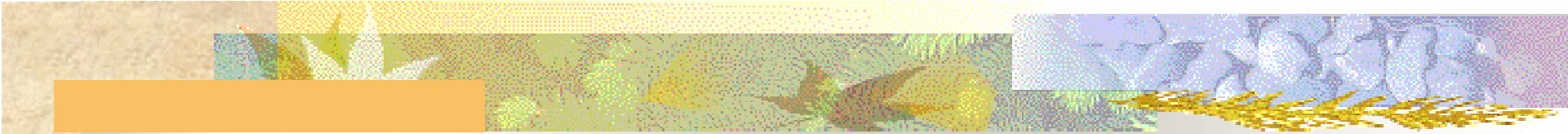
$$c = \rho p_i$$

$$p_i^* = \frac{1}{\rho} c \quad (2.22)$$


$$\begin{aligned}\pi(x) &= (p - c)x - F \\ &= \left(\frac{1}{\rho}c - c\right)x - F \\ &= \frac{1 - \rho}{\rho}cx - F\end{aligned}\tag{2.23}$$

◎ From the firm's profit function, we know that the **smaller** ρ , then the **lower** substitution among commodities, and the difference between each pair of commodities is **larger**, thus the firm can get the **higher** profit.

➡ Under the assumption of **free entry**, then the profit would be zero for the last entrant, thus,


$$\pi(x) = \frac{1-\rho}{\rho}cx - F = 0$$

$$\left(\frac{1-\rho}{\rho}\right)cx = F$$

$$x^* = \frac{1}{c} \frac{\rho}{1-\rho} F$$

$$= \frac{\rho}{1-\rho} \frac{F}{c} \quad (2.24)$$

$$\frac{\partial x^*}{\partial F} = \frac{\rho}{1 - \rho c} \frac{1}{c} > 0 \quad (2.25)$$

$$\frac{\partial x^*}{\partial c} = -\frac{\rho}{1 - \rho c^2} \frac{1}{c^2} < 0 \quad (2.26)$$

$$\begin{aligned} \frac{\partial x^*}{\partial \rho} &= \frac{(1 - \rho) - \rho(-1) F}{(1 - \rho)^2} \frac{1}{c} \\ &= \frac{1}{(1 - \rho)^2} \frac{F}{c} > 0 \end{aligned} \quad (2.27)$$


$$p^* = \frac{1}{\rho}c \quad (2.28)$$

$$\frac{\partial p^*}{\partial \rho} = -\frac{1}{\rho^2}c < 0 \quad (2.29)$$

- (i) the quantity produced by each firm is **increasing** ρ in ρ and F , but **decreasing** in c ,
- (ii) the price charged by each firm is **decreasing** in ρ .



◎ Under the symmetric assumption:

$$x_i = x \quad , \quad p_i = p \quad \text{for all } i$$

Then

$$x_0 + np x = I \quad (2.30)$$

$$np x = I - x_0$$

$$p^* = \frac{1}{\rho} c$$

$$x^* = \frac{\rho}{1 - \rho} \frac{F}{c} \quad (2.31)$$

$$\Rightarrow np_x = I - x_0$$

$$n \cdot \frac{c}{\rho} \cdot \frac{\rho}{1-\rho} \cdot \frac{F}{c} = I - x_0$$

$$n^* = \frac{1}{F} (1-\rho)(I - x_0) \quad (2.32)$$

$$\frac{\partial n^*}{\partial F} < 0 \quad (2.33)$$

$$\frac{\partial n^*}{\partial I} > 0 \quad (2.34)$$

$$\frac{\partial n^*}{\partial \rho} < 0 \quad (2.35)$$



2.3 The Dixit-Stiglitz Model of Monopolistic Competition

Assumption:

1. The economy has **two sectors**: **agriculture** sector and **manufacturing** sector.
2. Agriculture sector is **perfectly competitive** and produces a single, homogenous good.
3. Manufacturing sector **produces a large variety of differentiated good**.
4. Assume that there are **a large number of potential manufactured goods**, so many that the product space can be represented as continuous.

◎ Consumer behavior

$$\max_{A, m_i} U = M^u A^{1-u} \quad (2.36)$$

$$M = \left[\int_0^n m(i)^\rho di \right]^{\frac{1}{\rho}} \quad 0 < \rho < 1 \quad (2.37)$$


$$\text{s.t. } p^A \cdot A + \int_0^n p(i)m(i)di = Y \quad (2.38)$$

Solve:

First stage:

$$\min_{m(i)} \int_0^n p(i)m(i)di \quad (2.39)$$

$$\text{s.t.} \left[\int_0^n m(i)^\rho di \right]^{\frac{1}{\rho}} = M \quad (2.40)$$


$$L = \int_0^n p(i)m(i)di + \lambda \{ M - [\int_0^n m(i)^\rho di]^\rho \}^{\frac{1}{\rho}}$$

$$\frac{\partial L}{\partial m(i)} = p(i) - \lambda \cdot \frac{1}{\rho} [\int_0^n m(i)^\rho di]^{\frac{1}{\rho}-1} \cdot \rho \cdot m(i)^{\rho-1} = 0$$

$$\frac{\partial L}{\partial m(j)} = p(j) - \lambda \cdot \frac{1}{\rho} [\int_0^n m(i)^\rho di]^{\frac{1}{\rho}-1} \cdot \rho \cdot m(j)^{\rho-1} = 0$$

$$\frac{m(i)^{\rho-1}}{m(j)^{\rho-1}} = \frac{p(i)}{p(j)} \quad (2.41)$$

$$m(i)^{\rho-1} = m(j)^{\rho-1} \frac{p(i)}{p(j)}$$

$$m(i)^{1-\rho} = m(j)^{1-\rho} \frac{p(j)}{p(i)}$$

$$\Rightarrow m(i) = m(j) \left[\frac{p(j)}{p(i)} \right]^{\frac{1}{1-\rho}} \quad (2.42)$$

$$\left[\int_0^n m(i)^\rho di \right]^{\frac{1}{\rho}} = \left[\int_0^n m(j)^\rho \left[\frac{p(j)}{p(i)} \right]^{\frac{\rho}{1-\rho}} di \right]^{\frac{1}{\rho}}$$

$$= m(j) p(j)^{\frac{1}{1-\rho}} \left[\int_0^n \left[\frac{1}{p(i)} \right]^{\frac{\rho}{1-\rho}} di \right]^{\frac{1}{\rho}}$$

$$= m(j) p(j)^{\frac{1}{1-\rho}} \left[\int_0^n p(i)^{\frac{\rho}{\rho-1}} di \right]^{\frac{1}{\rho}} \quad (2.43)$$

$$M = m(j)p(j)^{\frac{1}{1-\rho}} \left[\int_0^n p(i)^{\frac{\rho}{\rho-1}} di \right]^{\frac{1}{\rho}} \quad (2.44)$$

$$m(j) = \frac{p(j)^{\frac{1}{\rho-1}}}{\left[\int_0^n p(i)^{\frac{\rho}{\rho-1}} di \right]^{\frac{1}{\rho}}} M \quad (2.45)$$

$$\begin{aligned} \int_0^n p(j)m(j)dj &= \frac{\int_0^n p(j) \cdot p(j)^{\frac{1}{\rho-1}} dj}{\left[\int_0^n p(i)^{\frac{\rho}{\rho-1}} di \right]^{\frac{1}{\rho}}} M \\ &= \frac{\left[\int_0^n p(j)^{\frac{\rho}{\rho-1}} dj \right]}{\left[\int_0^n p(i)^{\frac{\rho}{\rho-1}} di \right]^{\frac{1}{\rho}}} M \\ &= \left[\int_0^n p(i)^{\frac{\rho}{\rho-1}} di \right]^{\frac{\rho-1}{\rho}} \cdot M \end{aligned} \quad (2.46)$$

Define **price index for manufactured products** by G

$$G \equiv \left[\int_0^n p(i)^{\frac{\rho}{\rho-1}} di \right]^{\frac{\rho-1}{\rho}} = \left[\int_0^n p(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} \quad (2.47)$$

where

$$\rho = \frac{\sigma-1}{\sigma}, \quad \rho-1 = \frac{\sigma-1-\sigma}{\sigma} = \frac{-1}{\sigma}$$

$$\frac{\rho}{\rho-1} = \frac{\frac{\sigma-1}{\sigma}}{-\frac{1}{\sigma}} = -(\sigma-1) = 1-\sigma \quad (2.48)$$

$$\begin{aligned}
m(j) &= \frac{p(j)^{\frac{1}{\rho-1}}}{\left[\int_0^n p(i)^{\frac{1}{\rho-1}} di\right]^{\frac{1}{\rho}}} M \\
&= \frac{p(j)^{\frac{1}{\rho-1}}}{G^{\frac{1}{\rho-1}}} M \\
&= \left[\frac{p(j)}{G}\right]^{\frac{1}{\rho-1}} M = \left[\frac{p(j)}{G}\right]^{-\sigma} \cdot M
\end{aligned} \tag{2.49}$$

◎ Second stage:

$$\max_{M, A} U = M^u A^{1-u}$$


$$\text{s.t. } GM + p^A \cdot A = Y \quad (2.50)$$

$$L = M^u \cdot A^{1-u} + \lambda(Y - GM - p^A \cdot A)$$

$$\frac{\partial L}{\partial M} = \mu \frac{U}{M} - \lambda G = 0 \quad (2.51)$$

$$\frac{\partial L}{\partial A} = (1 - \mu) \frac{U}{A} - \lambda p^A = 0$$

$$\frac{\mu}{(1 - \mu)} \frac{A}{M} = \frac{G}{p^A} \quad (2.52)$$


$$MG = \frac{\mu}{(1-\mu)} p^A \cdot A$$

$$\frac{\mu}{(1-\mu)} p^A \cdot A + p^A \cdot A = Y$$

$$\left(\frac{\mu}{(1-\mu)} + 1\right) p^A \cdot A = Y$$

$$\frac{1}{(1-\mu)} p^A \cdot A = Y$$

$$A = \frac{1-\mu}{p^A} Y \quad (2.53)$$

$$M = \frac{1}{G} \frac{\mu}{1-\mu} (1-\mu) Y = \frac{\mu}{G} Y \quad (2.54)$$

$$\begin{aligned}
m(j) &= \left[\frac{p(j)}{G} \right]^{-\sigma} \cdot M \\
&= \left[\frac{p(j)}{G} \right]^{-\sigma} \cdot \frac{\mu}{G} Y \\
&= \mu Y \frac{p(j)^{-\sigma}}{G^{-(\sigma-1)}} \quad j \in [0, n] \quad (2.55)
\end{aligned}$$

$$\begin{aligned}
U &= M^u A^{1-u} \\
&= \left(\frac{\mu}{G} Y \right)^u \left(\frac{1-\mu}{p^A} Y \right)^{1-u} \\
&= \mu^\mu (1-\mu)^{1-u} Y G^{-\mu} (p^A)^{-(1-u)} \quad (2.56)
\end{aligned}$$

➔ indirect utility function

Under symmetric assumption: $p(i) = p(j) = p^M$

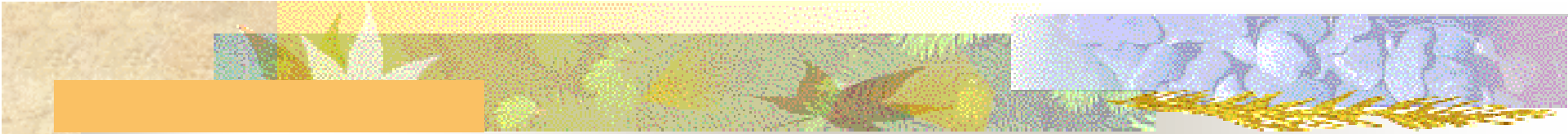
$$\begin{aligned} G &= \left[\int_0^n p(i)^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} \\ &= [np(i)^{1-\sigma}]^{\frac{1}{1-\sigma}} \\ &= n^{\frac{1}{1-\sigma}} p^M \end{aligned} \quad (2.57)$$



2.4 Dixit-Stiglitz Model with Spatial Context

2.4.1 Assumptions

- (1) The economy consists of a **finite set** of location (cities, regions or countries), let R denotes the number of locations.
- (2) Assume that each variety is produced in only one location, and that all varieties produced in a particular location are **symmetric**, having the same technology and price.
- (3) We denote the number of varieties produced in location γ by n_γ .

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- (4) We define the **mill** or **f.o.b** price of one of these varieties by p_γ^M .
- (5) Agricultural and manufactured goods can be shipped between locations and may incur transport cost in shipment.
- (6) The transport cost between location is specified by “**iceberg**” form, which is originated by Von Thünen and Paul Samuelson, and it is useful to avoid modeling a **separate transportation industry**.

2.4.2 The Model

◎The Manufacturing Price

$$p_{\gamma s}^M = p_{\gamma}^M T_{\gamma s}^M \quad (2.58)$$



where

p_{γ}^M : denotes that a manufacturing variety produced at location γ is sold at price p_{γ}^M (**mill price**).

$p_{\gamma s}^M$: is the **delivered (c.i.f.)** price of that variety be sold at each consumption location s .

$T_{\gamma s}^M$: represents the amount of manufactured good dispatched per unit received.

$1/T_{\gamma s}^M$: implies that the fraction of original a unit of any variety of manufactured goods is shipped from a location γ , and actually arrive to locations s .

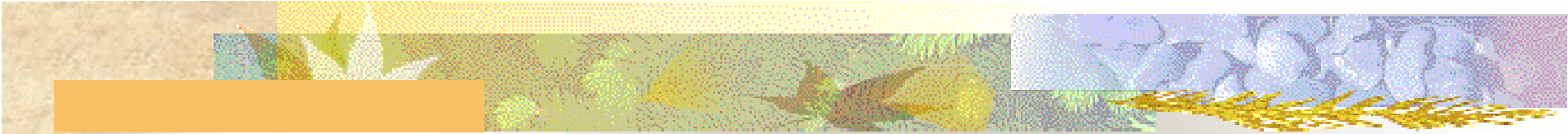


© The manufacturing price index in location s

Substituting (2.58) into (2.57), we have “**the manufacturing price index**” for location s as:

$$G_s = \left[\sum_{\gamma=1}^R n_{\gamma} (p_{\gamma}^M T_{\gamma s}^M)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad s = 1, 2, \dots, R \quad (2.59)$$

- ➔ please note that σ which may take a different value in each location.
- ➔ Assume that all varieties produced in a particular location have the same price.


- 
- ◎ The consumption demand in location s for a product produced in γ ,

Substituting (2.58) into (2.55), we have the consumption demand in location s for a product produced in γ as

$$m_s = \mu Y_s (p_\gamma^M T_{\gamma s}^M)^{-\sigma} G_s^{(\sigma-1)} \quad (2.60)$$

- ◎ The total sales of a single location γ for each variety as

$$\begin{aligned} q_\gamma^M &= \sum_{s=1}^R m_s \cdot T_{\gamma s}^M \\ &= u \sum_{s=1}^R Y_s (P_\gamma^M T_{\gamma s}^M)^{-\sigma} G_s^{(\sigma-1)} \cdot T_{\gamma s}^M \end{aligned} \quad (2.61)$$

- 
- ➔ The **total sales** for each variety, q_γ^M , depends on **income** in each location Y_s , the **price index** in each location G_s , **transportation costs** $T_{\gamma s}^M$, and the **mill price** p_γ^M .
 - ➔ Notice that because the **delivered prices** of the same variety at all consumption locations change proportionally to the mill price, and because each consumer's demand for a variety has a constant price elasticity σ , the elasticity of the aggregate demand for each variety with respect to its mill price is also σ , regardless of the spatial distribution of consumers.



◎ Producer Behavior

Agricultural Sector:

The agriculture good is produced only using labor with a **constant-returns technology** under conditions of **perfect competition**.

Manufacturing sector:

The manufacturing good is assumed to involve economics of scale which is arisen at the level of the variety, and the production of a quantity q^M of any variety at any given location is specified as:

$$l^M = F + c^M q^M \quad (2.62)$$



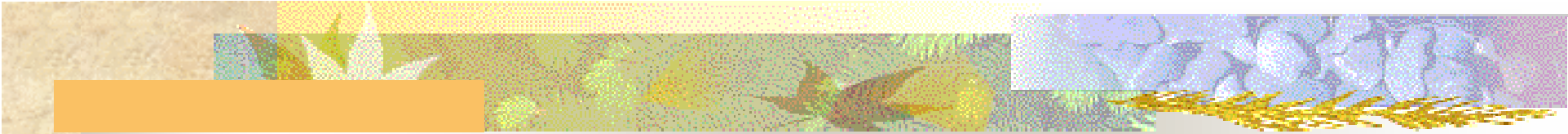
Where

l^M : denotes **total labor inputs** requirement,

F : **fixed labor inputs** requirement,

c^M : **marginal labor input** requirement

q^M : the quantity be produced for each variety

- 
- ➔ Production technology is the **same** for all varieties and in all locations.
 - ➔ The production of manufacturing good is assumed to **use labor as input** only.
 - ➔ Because of **increasing returns to scale**, consumer's preference for variety, and the unlimited number of potential varieties of manufactured goods, no firm will choose to produce the same variety supplied by another firm.
 - ➔ It implies that each variety is produced in only **one location**, by a **single, specialized** firm, so that the number of manufacturing firms in operation is the same as the number of available varieties.

2.4.3. The Analysis

◎ The profit maximization for each firm

Consider a particular firm producing a specific variety at location γ and facing a given wage rate, w_γ^M , for manufacturing workers there. Then, with a mill price p_γ^M , its **profit** is given by

$$\pi_\gamma = p_\gamma^M q_\gamma^M - w_\gamma^M (F + c^M q_\gamma^M), \quad (2.73)$$

where q_γ^M is given by the **total sales** for each variety at location γ , which is specified as (2.61).

$$\max_{p_\gamma^M} \pi_\gamma = p_\gamma^M q_\gamma^M - w_\gamma^M (F + c^M q_\gamma^M) \quad (2.64)$$

➔ Each firm is assumed to choose its price taking the price indices, G_s , as given.

$$\frac{\partial \pi_\gamma}{\partial p_\gamma^M} = q_\gamma^M + p_\gamma^M \frac{\partial q_\gamma^M}{\partial p_\gamma^M} - c^M w_\gamma^M \frac{\partial q_\gamma^M}{\partial p_\gamma^M} = 0$$

$$\Rightarrow q_\gamma^M + (p_\gamma^M - c^M w_\gamma^M) \frac{\partial q_\gamma^M}{\partial p_\gamma^M} = 0 \quad (2.75)$$

From (2.61), we have

$$\begin{aligned}
 \frac{\partial q_\gamma^M}{\partial p_\gamma^M} &= -\sigma \mu \sum_{s=1}^R Y_s (p_\gamma^M T_{\gamma s}^M)^{-\sigma-1} G_s^{\sigma-1} \cdot T_{\gamma s}^M \cdot T_{\gamma s}^M \\
 &= -\sigma \mu \sum_{s=1}^R Y_s (p_\gamma^M T_{\gamma s}^M)^{-\sigma-1} G_s^{\sigma-1} T_{\gamma s}^M \cdot T_{\gamma s}^M \\
 &= -\sigma \mu \sum_{s=1}^R Y_s (p_\gamma^M T_{\gamma s}^M)^{-\sigma} \cdot G_s^{\sigma-1} \cdot (p_\gamma^M)^{-1} \cdot T_{\gamma s}^M \\
 &= -\sigma \frac{\mu \sum_{s=1}^R Y_s (p_\gamma^M T_{\gamma s}^M)^{-\sigma} G_s^{\sigma-1} \cdot T_{\gamma s}^M}{p_\gamma^M} \\
 &= -\sigma \frac{q_\gamma^M}{p_\gamma^M} \tag{2.66}
 \end{aligned}$$

Substitute (2.66) into (2.65), we have

$$q_{\gamma}^M - (p_{\gamma}^M - c^M w_{\gamma}^M) \sigma \frac{q_{\gamma}^M}{p_{\gamma}^M} = 0$$

$$q_{\gamma}^M \left[1 - (p_{\gamma}^M - c^M w_{\gamma}^M) \frac{\sigma}{p_{\gamma}^M} \right] = 0$$

$$\frac{p_{\gamma}^M - c^M w_{\gamma}^M}{p_{\gamma}^M} \cdot \sigma = 1$$

$$p_{\gamma}^M - c^M w_{\gamma}^M = \frac{1}{\sigma} p_{\gamma}^M$$

$$p_{\gamma}^M \left(1 - \frac{1}{\sigma} \right) = c^M w_{\gamma}^M$$

$$p_{\gamma}^M = \frac{c^M w_{\gamma}^M}{1 - \frac{1}{\sigma}} = \frac{1}{\rho} c^M w_{\gamma}^M \quad \left(\text{Since, } \rho = \frac{\sigma - 1}{\sigma} \right) \quad (7.67)$$



- ① The Assumption of free entry

We assume that there is **free entry** and **exit** in response to **profits** or **losses**. Thus, substituting (2.67) into (2.63), we have

$$\pi_{\gamma} = p_{\gamma}^M q_{\gamma}^M - w_{\gamma}^M (F + c^M q_{\gamma}^M) = 0$$

$$\Rightarrow \pi_{\gamma} = \frac{\sigma}{\sigma - 1} c^M w_{\gamma}^M q_{\gamma}^M - w_{\gamma}^M F - c^M w_{\gamma}^M q_{\gamma}^M = 0$$

$$\Rightarrow \pi_{\gamma} = \left(\frac{\sigma}{\sigma - 1} - 1 \right) c^M w_{\gamma}^M q_{\gamma}^M - w_{\gamma}^M F = 0$$

$$\Rightarrow \pi_{\gamma} = \frac{1}{\sigma - 1} c^M w_{\gamma}^M q_{\gamma}^M - w_{\gamma}^M F = 0$$

$$\Rightarrow \pi_{\gamma} = w_{\gamma}^M \left(\frac{c^M q_{\gamma}^M}{\sigma - 1} - F \right) = 0 \quad (2.68)$$

◎ The equilibrium output for each firm

From (2.68), we can derive **the equilibrium output** for each firm (i.e., each variety) as

$$q^* = F(\sigma - 1) / c^M \quad (2.69)$$

◎ The equilibrium labor input

Substituting (2.69) into (2.62), we have **the equilibrium labor input** as

$$\begin{aligned} l^* &= F + c^M q^* \\ &= F + c^M [F(\sigma - 1) / c^M] \\ &= F + F(\sigma - 1) \\ &= F\sigma \end{aligned} \quad (2.70)$$

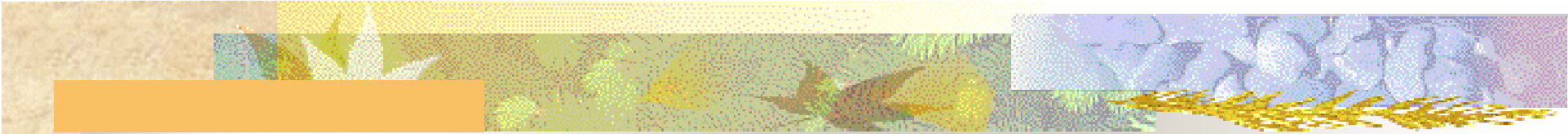
➔ Both q^* and l^* are **constants** common to every active firm in all locations, that is, the optimal q^* and l^* are independent of the regions where the firms locate.



◎ The equilibrium number of manufacturing firms

If L_γ^M is the number of manufacturing workers at location γ , and n_γ is **the number of manufacturing firms** (i.e., **the number of the varieties** produced) at γ , then

$$n_\gamma^* = L_\gamma^M / l^* = L_\gamma^M / F\sigma \quad (2.71)$$



→ The size of market (i.e., the size of consumption demand) affects neither the markup of price over marginal cost nor the scale at which individual goods are produced. [please refer both equations (2.67) and (2.69)]

→ All **scale effects** work through changes in the variety of goods available.

→ The Dixit-Stiglitz model implies that all **market-size effect** work through change in variety.

(The alternatives ways the economy takes advantage of the extent of the market is by producing at larger scale.)

◎ The Manufacturing Wage Equation

At each location γ , the **total sales** for every variety q_γ^M (2.61) is equivalent to the firm produce q^* (2.69), thus the following equation is satisfied:

$$q^* = \mu \sum_{s=1}^R Y_s (p_\gamma^M)^{-\sigma} (T_{\gamma^s}^M)^{1-\sigma} G_s^{\sigma-1} \quad (2.72)$$

$$(p_\gamma^M)^\sigma = \frac{\mu}{q^*} \sum_{s=1}^R Y_s (T_{\gamma^s}^M)^{1-\sigma} G_s^{\sigma-1} \quad (2.73)$$

Substituting (2.67) into (2.73), then we have

$$\left(\frac{\sigma}{\sigma-1} \cdot c^M w_\gamma^M\right)^\sigma = \frac{\mu}{q^*} \sum_{s=1}^R Y_s (T_{\gamma^s}^M)^{1-\sigma} G_s^{\sigma-1}$$

$$w_\gamma^M = \left(\frac{\sigma-1}{\sigma c^M}\right) \left[\frac{\mu}{q^*} \sum_{s=1}^R Y_s (T_{\gamma^s}^M)^{1-\sigma} G_s^{\sigma-1}\right]^{1/\sigma} \quad (2.74)$$



From (2.74), we know that

The wage at location γ is higher, if

- (1) the **incomes** in the firm's markets, Y_s , are higher,
- (2) the firm's **access** to these markets is better (lower $T_{\gamma s}^M$)
- (3) the firm faces less competition in these markets, from (2.59), we know $(n \downarrow \Rightarrow G_s \uparrow \Rightarrow w_\gamma^M \uparrow)$



◎ The real wage

From “**indirect utility function**” (2.56), we (2.75)

$$V_{\gamma} = \mu^{\mu} (1 - \mu)^{1 - \mu} Y_{\gamma} G_{\gamma}^{-\mu} (p_{\gamma}^A)^{-(1 - \mu)} \quad (2.75)$$

Where Y_{γ} : denotes the **nominal income** (or **wage**) in location γ ,

Thus, **the real wage** of manufacturing workers in location, denoted γ , is given by ω_{γ}^M as follows:

$$\omega_{\gamma}^M = w_{\gamma}^M G_{\gamma}^{-\mu} (p_{\gamma}^A)^{-(1 - \mu)} \quad (2.76)$$



$G_\gamma^\mu (p_\gamma^A)^{1-\mu}$: define **the cost-of –living index**,

And if we **normalize** the price of agricultural good as equal one, $p_r^A=1$,we can simplify **the cost-of –living index** as : G_γ^μ

In turn, **the real wage** and **the indirect utility** can be rewritten, respectively, as follows:

$$\omega_\gamma^M = w_\gamma^M G_\gamma^{-\mu} \quad (2.77)$$


$$V_\gamma = \mu^\mu (1-\mu)^{1-\mu} w_\gamma G_\gamma^{-\mu} \quad (2.78)$$



2.4.4 The Normalizations and Further Simplification

In order to simplify the previous equations, we can choose units such that the marginal labor requirement satisfies the following equation:

$$c^M = \frac{\sigma - 1}{\sigma} \quad (2.79)$$



Substituting (2.78) into (2.69), (2.70), and (2.71), we have the results that

$$p_{\gamma}^M = w_{\gamma}^M \quad (2.80)$$

$$q^* = l^* = \mu \quad (2.81)$$

$$F = \mu / \sigma \quad (2.82)$$

$$n_{\gamma} = L_{\gamma}^M / \mu \quad (2.83)$$

Using equations (2.80)-(2.83), **the manufacturing price index**

G_{γ} [shown as in equation (2.59)], and **the nominal wage**

w_{γ}^M [shown as in equation (2.74)], are given, respectively, by

$$\begin{aligned}
\mathbf{G}_\gamma &= \left[\sum_{s=1}^R n_s (p_s^M T_{s\gamma}^M)^{1-\sigma} \right]^{1/(1-\sigma)} \\
&= \left[\frac{1}{\mu} \sum_{s=1}^R L_s^M (w_s^M T_{s\gamma}^M)^{1-\sigma} \right]^{1/(1-\sigma)}
\end{aligned} \tag{2.84}$$

$$\begin{aligned}
w_\gamma^M &= \left(\frac{\sigma-1}{\sigma c^M} \right) \left[\frac{\mu}{q^*} \sum_{s=1}^R Y_s (T_{s\gamma}^M)^{1-\sigma} G_s^{\sigma-1} \right]^{1/\sigma} \\
&= \left[\sum_{s=1}^R Y_s (T_{s\gamma}^M)^{1-\sigma} G_s^{\sigma-1} \right]^{1/\sigma}
\end{aligned} \tag{2.85}$$

➡ We would use these two equations to examine both “**the equilibrium of core and periphery**” and “**the equilibrium of symmetry**”, and also to investigate its stability on next chapter.