Lecture 2

Monopolistic Competition and Optimum Product Differentiation

Reference:

Dixit and Stiglitz, 1977, Monopolistic Competition and Optimum Product Diversity, *American Economic Review* 67, 297-308.

2.1 Pricing Behavior

The utility function for consumers

$$u = u(x_0, v(x_1, x_2, \ldots\ldots x_n)) \qquad (2.1)$$

Assumption:

(1) A separable utility function with convex indifference surfaces.

(2) V is a symmetric function, thus all commodities have equal fixed and marginal costs.

(3) Commodities with a pair close together have better mutual substitutes than a pair farther apart.

(4) u can be regarded as representing Samuelsonian social indifference curves, or regarded as a multiple of a representative consumer's utility.

(5) Product diversity can be interpreted either as different consumers using different varieties, or as diversification on the part of each consumer.

⦿ The Characteristics of Monopolistic Competition

(1)The number of firms is large enough.

(2)The commodities produced by each firm are good substitutes among themselves.

(3) Firms enter the market until the next potential entrant would make a loss.

(4) Each firm has not the dominant power to determine the commodity’s price, it is also not as perfect competition market to take the price as given variable (i.e., exogenously determined)

⦿Utility Function:

$$u = u(x_0, [\sum_{i=1} x_i^{\rho}]^{\frac{1}{\rho}}) \equiv x_0 + (\sum_{i=1}^{n} x_i^{\rho})^{\frac{1}{\rho}} \qquad (2.2)$$

(i) For concavity, we need $\rho < 1$

(ii) For allowing a situation where several of the x_i are zero, we need $\rho > 0$

(iii) ρ denotes the substitution index among commodities, $\rho \in (0,1)$, it also interprets the preference of diversity commodities for the consumers, the smaller of the ρ , implies the consumers prefer to consume the commodities more diversity.

(iv) u is a homothetic function in its arguments.

$$\underset{x_0, x_i}{Max}\ U(x_0, [\sum_{i=1} x_i^{\rho}]^{\frac{1}{\rho}}) \qquad (2.3)$$

s.t

$$x_0 + \sum_{i=1}^{n} p_i x_i = I \qquad (2.4)$$

Lagrange function:

$$L = x_0 + (\sum_{i=1}^{n} x_i^{\rho})^{\frac{1}{\rho}} + \lambda(I - \sum_{i=1}^{n} p_i x_i - x_0) \qquad (2.5)$$

λ : Lagrange parameter, it denotes the marginal contribution of income to utility.

⦿Budget Constraint:

$$x_0 + \sum_{i=1}^{n} p_i x_i = I \qquad (2.6)$$

$$\frac{\partial L}{\partial x_0} = 1 - \lambda = 0$$

$$\lambda = 1$$

$$\frac{\partial L}{\partial x_i} = \frac{1}{\rho}\left(\sum_{i=1}^{n} x_i^{\rho}\right)^{\frac{1}{\rho}-1} \cdot \rho x_i^{\rho-1} - \lambda p_i = 0$$

$$\Rightarrow \left(\sum_{i=1}^{n} x_i^{\rho}\right)^{\frac{1}{\rho}-1} x_i^{\rho-1} = \lambda p_i$$

$$\Rightarrow \left(\sum_{i=1}^{n} x_i^{\rho}\right)^{\frac{1-\rho}{\rho}} x_i^{\rho-1} = \lambda p_i$$

$$\Rightarrow \left[\left(\sum_{i=1}^{n} x_i^{\rho}\right)^{\frac{1}{\rho}}\right]^{1-\rho} x_i^{\rho-1} = \lambda p_i \qquad (2.7)$$

$$\Rightarrow \left[\left(\sum_{i=1}^{n} x_i^{\rho}\right)^{\frac{1}{\rho}}\right]^{1-\rho} x_i^{\rho-1} = p_i \qquad (2.8)$$

Therefore, we have:

$$(V)^{1-\rho} x_i^{\rho-1} = p_i \qquad (2.8')$$

In turn:

$$x_i = (p_i)^{\frac{1}{\rho-1}} V \qquad (2.9)$$

Where: $V \equiv (\sum_{i=1}^{n} x_i^{\rho})^{\frac{1}{\rho}}$

$$\frac{\partial x_i}{\partial p_i} = \frac{1}{\rho - 1} p_i^{\frac{1}{\rho-1}-1} V$$

$$= \frac{1}{\rho - 1} \frac{p_i^{\frac{1}{\rho-1}}}{p_i} V \qquad (2.10)$$

$$= \frac{1}{\rho - 1} \frac{x_i}{p_i}$$

$$\varepsilon = -\frac{\frac{\partial x_i}{x_i}}{\frac{\partial p_i}{p_i}} = -\frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i}$$

$$= -\frac{1}{\rho - 1}\frac{x_i}{p_i}\frac{p_i}{x_i}$$

$$= \frac{1}{1-\rho} \qquad (2.11)$$

➡the higher ρ, the larger ε, it implies the close substitution among commodities, that is , the difference between a pair of commodities become smaller.

⦿ If we assume there are symmetric across the commodities x_i , that is, the consumers buy each commodity with the same amount, then the utility function can be simplified as:

$$V = (\sum_{i=1}^{n} x_i^{\rho})^{\frac{1}{\rho}}$$

$$= (nx^{\rho})^{\frac{1}{\rho}}$$

$$= n^{\frac{1}{\rho}} x \qquad (7.12) \qquad \Rightarrow V_e = \ln V = \frac{1}{\rho}\ln n + \ln x$$

$$V = e^{Ve} = e^{\ln n^{\frac{1}{\rho}} x} \qquad (2.13)$$

(i) $$\frac{\partial \boldsymbol{V}}{\partial \rho} = \boldsymbol{e}^{\boldsymbol{Ve}} \cdot \frac{\partial \boldsymbol{V_l}}{\partial \rho} = \boldsymbol{e}^{\ln \boldsymbol{n}^{\frac{1}{\rho}} \boldsymbol{x}} \cdot \left(-\frac{1}{\rho^2}\right) \ln \cdot \boldsymbol{n}$$

$$= -\frac{1}{\boldsymbol{e}^2} \boldsymbol{n}^{\frac{1}{\rho}} \cdot \boldsymbol{x} \cdot \ln \boldsymbol{n} < 0 \qquad (2.14)$$

(ii) $$\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{n}} = \frac{1}{\rho} \boldsymbol{n}^{\frac{1}{\rho}-1} \cdot \boldsymbol{x} = \frac{1}{\rho} \boldsymbol{n}^{\frac{1-\rho}{\rho}} \cdot \boldsymbol{x} > 0 \qquad (2.15)$$

- Given $\boldsymbol{n}$, the smaller ρ implies consumers prefer to consume commodities more diversity, and thus the smaller ρ among commodities, the higher utility level for the consumers enjoying.

- Given ρ the large number of $\boldsymbol{n}$, the higher the utility level can achieved.

Firm's Profit function:

$$\pi(x_i) = p_i x_i - c x_i - F \qquad (2.16)$$

$$\max_{x_i} \pi(x_i) \qquad \frac{\partial \pi(x_i)}{\partial x_i} = p_i + x_i \frac{\partial p_i}{\partial x_i} - c$$

$$= p_i + x_i \frac{1}{\frac{\partial x_i}{\partial p_i}} - c$$

$$= p_i + x_i \frac{1}{\frac{1}{\rho - 1} \frac{x_i}{p_i}} - c$$

$$= p_i + (\rho - 1) p_i - c = 0$$

$$\rho p_i = c$$

$$p_i^* = \frac{1}{\rho} c \qquad (2.17)$$

S.O.C

$$\frac{\partial^2 \pi}{\partial x_i^2} = \frac{\partial p_i}{\partial x_i} + \frac{\partial p_i}{\partial x_i} + x_i \frac{\partial^2 p_i}{\partial x_i^2}$$

$$= 2\frac{\partial p_i}{\partial x_i} + x_i \frac{\partial^2 p_i}{\partial x_i^2}$$

$$= 2\frac{1}{\frac{\partial x_i}{\partial p_i}} + x_i \frac{\partial^2 p_i}{\partial x_i^2}$$

$$= 2\frac{1}{\frac{1}{\rho - 1}\frac{x_i}{p_i}} + x \frac{\partial^2 p_i}{\partial x_i^2}$$

$$= 2\frac{(\rho - 1) p_i}{x_i} + x_i \frac{\rho - 1}{x_i}[-\frac{p_i}{x_i} + \frac{\partial p_i}{\partial x_i}]$$

$$= 2\frac{(\rho-1)\boldsymbol{p_i}}{\boldsymbol{x_i}} + (\rho-1)[-\frac{\boldsymbol{p_i}}{\boldsymbol{x_i}} + (\rho-1)\frac{\boldsymbol{p_i}}{\boldsymbol{x_i}}]$$

$$= 2\frac{(\rho-1)\boldsymbol{p_i}}{\boldsymbol{x_i}} + (\rho-1)\frac{\boldsymbol{p_i}}{\boldsymbol{x_i}}(\rho-2)$$

$$= (\rho-1)\frac{\boldsymbol{p_i}}{\boldsymbol{x_i}}[2+\rho-2]$$

$$= \rho(\rho-1)\frac{\boldsymbol{p_i}}{\boldsymbol{x_i}} < 0, \qquad (2.18)$$

where

$$\frac{\partial \boldsymbol{p_i}}{\partial \boldsymbol{x_i}} = (\rho-1)\frac{\boldsymbol{p_i}}{\boldsymbol{x_i}} \qquad (2.19)$$

$$\frac{\partial^2 p_i}{\partial x_i^2} = -(\rho-1)\frac{p_i}{x_i^2} + (\rho-1)\frac{1}{x_i}\frac{\partial p_i}{\partial x_i} \qquad (2.20)$$

$$\max_{p_i} \pi_i \qquad \frac{\partial \pi_i}{\partial p_i} = x_i + p_i \frac{\partial x_i}{\partial p_i} - c\frac{\partial x_i}{\partial p_i} = 0$$

$$\Rightarrow x_i + (p-c)(\frac{\partial x_i}{\partial p_i}) = 0$$

$$x_i = (p-c)(-\frac{\partial x_i}{\partial p_i})$$

$$= (p-c)(-\frac{1}{\rho - 1}\frac{x_i}{p_i})$$

$$= (p-c)(\frac{1}{1-\rho})\frac{x_i}{p_i} \qquad (2.21)$$

$$1=\left(\frac{1}{1-\rho}\right)\frac{\boldsymbol{p_i}-\boldsymbol{c}}{\boldsymbol{p_i}}$$

$$(1-\rho)=\frac{\boldsymbol{p_i}-\boldsymbol{c}}{\boldsymbol{p_i}}$$

$$(1-\rho)\boldsymbol{p_i}=\boldsymbol{p_i}-\boldsymbol{c}$$

$$\boldsymbol{c}=[1-(1-\rho)]\boldsymbol{p_i}$$

$$\boldsymbol{c}=\rho\boldsymbol{p_i}$$

$$\boldsymbol{p_i^*}=\frac{1}{\rho}\boldsymbol{c} \qquad (2.22)$$

$$\pi(x) = (p - c)x - F$$

$$= (\frac{1}{\rho}c - c)x - F$$

$$= \frac{1-\rho}{\rho}cx - F \qquad (2.23)$$

◉ From the firm's profit function, we know that the smaller ρ , then the lower substitution among commodities, and the difference between each pair of commodities is larger, thus the firm can get the higher profit.

➡Under the assumption of free entry, then the profit would be zero for the last entrant, thus,

$$\pi(x) = \frac{1-\rho}{\rho} cx - F = 0$$

$$(\frac{1-\rho}{\rho}) cx = F$$

$$x^* = \frac{1}{c} \frac{\rho}{1-\rho} F$$

$$= \frac{\rho}{1-\rho} \frac{F}{c} \qquad (2.24)$$

$$\frac{\partial \boldsymbol{x}^*}{\partial \boldsymbol{F}} = \frac{\rho}{1-\rho}\frac{1}{\boldsymbol{c}} > 0 \qquad (2.25)$$

$$\frac{\partial \boldsymbol{x}^*}{\partial \boldsymbol{c}} = -\frac{\rho}{1-\rho}\frac{1}{\boldsymbol{c}^2} < 0 \qquad (2.26)$$

$$\frac{\partial \boldsymbol{x}^*}{\partial \rho} = \frac{(1-\rho)-\rho(-1)}{(1-\rho)^2}\frac{\boldsymbol{F}}{\boldsymbol{c}}$$

$$= \frac{1}{(1-\rho)^2}\frac{\boldsymbol{F}}{\boldsymbol{c}} > 0 \qquad (2.27)$$

$$\boldsymbol{p}^* = \frac{1}{\rho}\boldsymbol{c} \qquad (2.28)$$

$$\frac{\partial \boldsymbol{p}^*}{\partial \rho} = -\frac{1}{\rho^2}\boldsymbol{c} < 0 \qquad (2.29)$$

(i) the quantity produced by each firm is increasing

ρ in and F, but decreasing in c,

(ii) the price charged by each firm is decreasing in ρ .

- Under the symmetric assumption:

$$x_i = x \;,\quad p_i = p \quad \text{for all } i$$

Then

$$x_0 + npx = I \tag{2.30}$$

$$npx = I - x_0$$

$$p^* = \frac{1}{\rho} c$$

$$x^* = \frac{\rho}{1-\rho} \frac{F}{c} \tag{2.31}$$

$$\Rightarrow \boldsymbol{npx} = \boldsymbol{I} - \boldsymbol{x}_0$$

$$\boldsymbol{n} \cdot \frac{\boldsymbol{c}}{\rho} \cdot \frac{\rho}{1-\rho} \cdot \frac{\boldsymbol{F}}{\boldsymbol{c}} = \boldsymbol{I} - \boldsymbol{x}_0$$

$$\boldsymbol{n}^* = \frac{1}{\boldsymbol{F}}(1-\rho)(\boldsymbol{I} - \boldsymbol{x}_0) \qquad (2.32)$$

$$\frac{\partial \boldsymbol{n}^*}{\partial \boldsymbol{F}} < 0 \qquad (2.33)$$

$$\frac{\partial \boldsymbol{n}^*}{\partial \boldsymbol{I}} > 0 \qquad (2.34)$$

$$\frac{\partial \boldsymbol{n}^*}{\partial \rho} < 0 \qquad (2.35)$$

2.3 The Dixit-Stiglitz Model of Monopolistic Competition

Assumption:

1. The economy has two sectors: agriculture sector and manufacturing sector.
2. Agriculture sector is perfectly competitive and produces a single, homogenous good.
3. Manufacturing sector produces a large variety of differentiated good.
4. Assume that there are a large number of potential manufactured goods, so many that the product space can be represented as continuous.

⦿Consumer behavior

$$\max_{A,m_i} U = M^u A^{1-u} \qquad (2.36)$$

$$\boldsymbol{M} = [\int_0^{\boldsymbol{n}} \boldsymbol{m}(\boldsymbol{i})^{\rho} \boldsymbol{di}]^{\frac{1}{\rho}} \qquad 0 < \rho < 1 \qquad (2.37)$$

$$\text{s.t.} \quad \boldsymbol{p}^{\mathrm{A}} \cdot \boldsymbol{A} + \int_0^{\boldsymbol{n}} \boldsymbol{p}(\boldsymbol{i})\boldsymbol{m}(\boldsymbol{i})\boldsymbol{di} = \boldsymbol{Y} \qquad (2.38)$$

Solve:

First stage:

$$\min_{\boldsymbol{m}(\boldsymbol{i})} \int_0^{\boldsymbol{n}} \boldsymbol{p}(\boldsymbol{i})\boldsymbol{m}(\boldsymbol{i})\boldsymbol{di} \qquad (2.39)$$

$$\text{s.t.}[\int_0^{\boldsymbol{n}} \boldsymbol{m}(\boldsymbol{i})^{\rho} \boldsymbol{di}]^{\frac{1}{\rho}} = \boldsymbol{M} \qquad (2.40)$$

$$L=\int_0^n p(i)m(i)di+\lambda\{M-[\int_0^n m(i)^\rho di]^{\frac{1}{\rho}}\}$$

$$\frac{\partial L}{\partial m(i)}=p(i)-\lambda\cdot\frac{1}{\rho}[\int_0^n m(i)^\rho di]^{\frac{1}{\rho}-1}\cdot\rho\cdot m(i)^{\rho-1}=0$$

$$\frac{\partial L}{\partial m(j)}=p(j)-\lambda\cdot\frac{1}{\rho}[\int_0^n m(i)^\rho di]^{\frac{1}{\rho}-1}\cdot\rho\cdot m(j)^{\rho-1}=0$$

$$\frac{m(i)^{\rho-1}}{m(j)^{\rho-1}}=\frac{p(i)}{p(j)} \qquad (2.41)$$

$$m(i)^{\rho-1} = m(j)^{\rho-1}\frac{p(i)}{p(j)}$$

$$m(i)^{1-\rho} = m(j)^{1-\rho}\frac{p(j)}{p(i)}$$

$$\Rightarrow m(i) = m(j)[\frac{p(j)}{p(i)}]^{\frac{1}{1-\rho}} \qquad (2.42)$$

$$[\int_0^n m(i)^\rho di]^{\frac{1}{\rho}} = [\int_0^n m(j)^\rho [\frac{p(j)}{p(i)}]^{\frac{\rho}{1-\rho}} di]^{\frac{1}{\rho}}$$

$$= m(j)p(j)^{\frac{1}{1-\rho}}[\int_0^n [\frac{1}{p(i)}]^{\frac{\rho}{1-\rho}} di]^{\frac{1}{\rho}}$$

$$= m(j)p(j)^{\frac{1}{1-\rho}}[\int_0^n p(i)^{\frac{\rho}{\rho-1}} di]^{\frac{1}{\rho}} \qquad (2.43)$$

$$M = m(j)\,p(j)^{\frac{1}{1-\rho}}\left[\int_0^n p(i)^{\frac{\rho}{\rho-1}}di\right]^{\frac{1}{\rho}} \qquad (2.44)$$

$$m(j) = \frac{p(j)^{\frac{1}{\rho-1}}}{\left[\int_0^n p(i)^{\frac{\rho}{\rho-1}}di\right]^{\frac{1}{\rho}}}M \qquad (2.45)$$

$$\int_0^n p(j)m(j)dj = \frac{\int_0^n p(j).p(j)^{\frac{1}{\rho-1}}dj}{\left[\int_0^n p(i)^{\frac{\rho}{\rho-1}}di\right]^{\frac{1}{\rho}}}M$$

$$= \frac{\left[\int_0^n p(j)^{\frac{\rho}{\rho-1}}dj\right]}{\left[\int_0^n p(i)^{\frac{\rho}{\rho-1}}di\right]^{\frac{1}{\rho}}}M$$

$$= \left[\int_0^n p(i)^{\frac{\rho}{\rho-1}}di\right]^{\frac{\rho-1}{\rho}} \cdot M \qquad (2.46)$$

Define price index for manufactured products by G

$$G \equiv \left[\int_0^n p(i)^{\frac{\rho}{\rho-1}} di\right]^{\frac{\rho-1}{\rho}} = \left[\int_0^n p(i)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \qquad (2.47)$$

where

$$\rho = \frac{\sigma-1}{\sigma}, \qquad \rho - 1 = \frac{\sigma - 1 - \sigma}{\sigma} = \frac{-1}{\sigma}$$

$$\frac{\rho}{\rho-1} = \frac{\frac{\sigma-1}{\sigma}}{-\frac{1}{\sigma}} = -(\sigma-1) = 1-\sigma \qquad (2.48)$$

$$m(j) = \frac{p(j)^{\frac{1}{\rho-1}}}{[\int_0^n p(i)^{\frac{\rho}{\rho-1}} di]^{\frac{1}{\rho}}} M$$

$$= \frac{p(j)^{\frac{1}{\rho-1}}}{G^{\frac{1}{\rho-1}}} M$$

$$= [\frac{p(j)}{G}]^{\frac{1}{\rho-1}} M = [\frac{p(j)}{G}]^{-\sigma} \cdot M \qquad (2.49)$$

⦿Second stage:

$$\max_{M,A} U = M^{u} A^{1-u}$$

$$\text{s.t.} \quad \boldsymbol{GM} + \boldsymbol{p}^{\mathrm{A}} \cdot \boldsymbol{A} = \boldsymbol{Y} \qquad (2.50)$$

$$\boldsymbol{L} = \boldsymbol{M}^{\boldsymbol{u}} \cdot \boldsymbol{A}^{1-\boldsymbol{u}} + \lambda(\boldsymbol{Y} - \boldsymbol{GM} - \boldsymbol{p}^{\mathrm{A}} \cdot \boldsymbol{A})$$

$$\frac{\partial L}{\partial M} = \mu \frac{U}{M} - \lambda G = 0 \qquad (2.51)$$

$$\frac{\partial L}{\partial A} = (1-\mu)\frac{U}{A} - \lambda p^{A} = 0$$

$$\frac{\mu}{(1-\mu)} \frac{\boldsymbol{A}}{\boldsymbol{M}} = \frac{\boldsymbol{G}}{\boldsymbol{p}^{\boldsymbol{A}}} \qquad (2.52)$$

$$MG = \frac{\mu}{(1-\mu)} p^A \cdot A$$

$$\frac{\mu}{(1-\mu)} p^A \cdot A + p^A \cdot A = Y$$

$$\left(\frac{\mu}{(1-\mu)} + 1\right) p^A \cdot A = Y$$

$$\frac{1}{(1-\mu)} p^A \cdot A = Y$$

$$A = \frac{1-\mu}{p^A} Y \qquad (2.53)$$

$$M = \frac{1}{G} \frac{\mu}{1-\mu} (1-\mu) Y = \frac{\mu}{G} Y \qquad (2.54)$$

$$m(j) = [\frac{p(j)}{G}]^{-\sigma} \cdot M$$

$$= [\frac{p(j)}{G}]^{-\sigma} \cdot \frac{\mu}{G} Y$$

$$= \mu Y \frac{p(j)}{G^{-(\sigma-1)}}^{-\sigma} \qquad j \in [0, n] \qquad (2.55)$$

$$U = M^u A^{1-u}$$

$$= (\frac{\mu}{G} Y)^u (\frac{1-\mu}{p^A} Y)^{1-u}$$

$$= \mu^{\mu} (1-\mu)^{1-u} Y G^{-\mu} (p^A)^{-(1-u)} \qquad (2.56)$$

➡ indirect utility function

Under symmetric assumption: $p(i) = p(j) = p^M$

$$G = [\int_0^n p(i)^{1-\sigma} di]^{\frac{1}{1-\sigma}}$$

$$= [np(i)^{1-\sigma}]^{\frac{1}{1-\sigma}}$$

$$= n^{\frac{1}{1-\sigma}} p^M \qquad (2.57)$$

2.4 Dixit-Stiglitz Model with Spatial Context

2.4.1 Assumptions

(1) The economy consists of a finite set of location (cities, regions or countries), let R denotes the number of locations.

(2) Assume that each variety is produced in only one location, and that all varieties produced in a particular location are symmetric, having the same technology and price.

(3) We denote the number of varieties produced in location γ by n_γ .

(4) We define the mill or f.o.b price of one of these varieties by p_{γ}^{M} .

(5) Agricultural and manufactured goods can be shipped between locations and may incur transport cost in shipment.

(6) The transport cost between location is specified by "**iceberg**" form, which is originated by Von Thünen and Paul Samuelson, and it is useful to avoid modeling a separate transportation industry.

2.4.2 The Model

- The Manufacturing Price

$$\boldsymbol{p}_{\gamma s}^{M} = \boldsymbol{p}_{\gamma}^{M} \boldsymbol{T}_{\gamma s}^{M} \qquad (2.58)$$

where

p_{γ}^{M} : denotes that a manufacturing variety produced at location γ is sold at price p_{γ}^{M} (mill price).

$p_{\gamma s}^{M}$: is the delivered (c.i.f.) price of that variety be sold at each consumption location s.

$T_{\gamma s}^{M}$: represents the amount of manufactured good dispatched per unit received.

$1/T_{\gamma s}^{M}$: implies that the fraction of original a unit of any variety of manufactured goods is shipped from a location γ, and actually arrive to locations s.

⦿ The manufacturing price index in location s

Substituting (2.58) into (2.57), we have "the manufacturing price index" for location s as:

$$G_s = [\sum_{\gamma=1}^{R} n_\gamma (p_\gamma^M T_{\gamma s}^M)^{1-\sigma}]^{\frac{1}{1-\sigma}} \quad s = 1,2,......R \qquad (2.59)$$

- please note that which may take a different value in each location.
- Assume that all varieties produced in a particular location have the same price.

- The consumption demand in location *s* for a product produced in γ ,

    Substituting (2.58) into (2.55), we have the consumption demand in location *s* for a product produced in γ as

$$m_s = \mu Y_s (p_\gamma^M T_{\gamma_s}^M)^{-\sigma} G_s^{(\sigma-1)} \qquad (2.60)$$

- The total sales of a single location γ for each variety as

$$q_\gamma^M = \sum_{s=1}^{R} m_s \cdot T_{\gamma s}^M$$

$$= u \sum_{s=1}^{R} Y_s (P_\gamma^M T_{\gamma s}^M)^{-\sigma} G_s^{(\sigma-1)} \cdot T_{\gamma s}^M \qquad (2.61)$$

- The total sales for each variety, q_γ^M, depends on income in each location Y_s, the price index in each location G_s, transportation costs $T_{\gamma s}^M$, and the mill price p_γ^M.
- Notice that because the **delivered prices** of the same variety at all consumption locations change proportionally to the mill price, and because each consumer's demand for a variety has a constant price elasticity σ, the elasticity of the aggregate demand for each variety with respect to its mill price is also σ, regardless of the spatial distribution of consumers.

⦿Producer Behavior

Agricultural Sector:

The agriculture good is produced only using labor with a constant-returns technology under conditions of perfect competition.

Manufacturing sector:

The manufacturing good is assumed to involve economics of scale which is arisen at the level of the variety, and the production of a quantity q^M of any variety at any given location is specified as:

$$l^M = F + c^M q^M \tag{2.62}$$

Where

l^M : denotes total labor inputs requirement,

F : fixed labor inputs requirement,

c^M : marginal labor input requirement

q^M : the quantity be produced for each variety

- Production technology is the same for all varieties and in all locations.
- The production of manufacturing good is assumed to use labor as input only.
- Because of increasing returns to scale, consumer's preference for variety, and the unlimited number of potential varieties of manufactured goods, no firm will choose to produce the same variety supplied by another firm.
- It implies that each variety is produced in only one location, by a single, specialized firm, so that the number of manufacturing firms in operation is the same as the number of available varieties.

2.4.3. **The Analysis**

- The profit maximization for each firm

Consider a particular firm producing a specific variety at location γ and facing a given wage rate, w_γ^M, for manufacturing workers there. Then, with a mill price p_γ^M, its profit is given by

$$\pi_\gamma = p_\gamma^M q_\gamma^M - w_\gamma^M (F + c^M q_\gamma^M), \qquad (2.73)$$

where q_γ^M is given by the **total sales** for each variety at location γ, which is specified as (2.61).

$$\max_{p_\gamma^M} \pi_\gamma = p_\gamma^M q_\gamma^M - w_\gamma^M (F + c^M q_\gamma^M) \qquad (2.64)$$

➡ Each firm is assumed to choose its price taking the price indices, G_s, as given.

$$\frac{\partial \pi_\gamma}{\partial p_\gamma^M} = q_\gamma^M + p_\gamma^M \frac{\partial q_\gamma^M}{\partial p_\gamma^M} - c^M w_\gamma^M \frac{\partial q_\gamma^M}{\partial p_\gamma^M} = 0$$

$$\Rightarrow q_\gamma^M + (p_\gamma^M - c^M w_\gamma^M) \frac{\partial q_\gamma^M}{\partial p_\gamma^M} = 0 \qquad (2.75)$$

From (2.61), we have

$$\frac{\partial q_{\gamma}^{M}}{\partial p_{\gamma}^{M}} = -\sigma\mu\sum_{s=1}^{R} Y_s (p_{\gamma}^{M} T_{\gamma s}^{M})^{-\sigma-1} G_s^{\sigma-1} \cdot T_{\gamma s}^{M} \cdot T_{\gamma s}^{M}$$

$$= -\sigma\mu\sum_{s=1}^{R} Y_s (p_{\gamma}^{M} T_{\gamma s}^{M})^{-\sigma-1} G_s^{\sigma-1} T_{\gamma s}^{M} \cdot T_{\gamma s}^{M}$$

$$= -\sigma\mu\sum_{s=1}^{R} Y_s (p_{\gamma}^{M} T_{\gamma s}^{M})^{-\sigma} \cdot G_s^{\sigma-1} \cdot (p_{\gamma}^{M})^{-1} \cdot T_{\gamma s}^{M}$$

$$= -\sigma \frac{\mu\sum_{s=1}^{R} Y_s (p_{\gamma}^{M} T_{\gamma s}^{M})^{-\sigma} G_s^{\sigma-1} \cdot T_{\gamma s}^{M}}{p_{\gamma}^{M}}$$

$$= -\sigma \frac{q_{\gamma}^{M}}{p_{\gamma}^{M}} \qquad (2.66)$$

Substitute (2.66) into (2.65), we have

$$q_\gamma^M - (p_\gamma^M - c^M w_\gamma^M)\sigma \frac{q_\gamma^M}{p_\gamma^M} = 0$$

$$q_\gamma^M [1 - (p_\gamma^M - c^M w_\gamma^M)\frac{\sigma}{p_\gamma^M}] = 0$$

$$\frac{p_\gamma^M - c^M w_\gamma^M}{p_\gamma^M} \cdot \sigma = 1$$

$$p_\gamma^M - c^M w_\gamma^M = \frac{1}{\sigma} p_\gamma^M$$

$$p_\gamma^M (1 - \frac{1}{\sigma}) = c^M w_\gamma^M$$

$$p_\gamma^M = \frac{c^M w_\gamma^M}{1 - \frac{1}{\sigma}} = \frac{1}{\rho} c^M w_\gamma^M \qquad \text{(Since, } \rho = \frac{\sigma - 1}{\sigma}\text{)} \quad (7.67)$$

The Assumption of free entry

We assume that there is free entry and exit in response to profits or losses. Thus, substituting (2.67) into (2.63), we have

$$\pi_\gamma = \boldsymbol{p}_\gamma^M \boldsymbol{q}_\gamma^M - \boldsymbol{w}_\gamma^M (\boldsymbol{F} + \boldsymbol{c}^M \boldsymbol{q}_\gamma^M) = 0$$

$$\Rightarrow \pi_\gamma = \frac{\sigma}{\sigma - 1} \boldsymbol{c}^M \boldsymbol{w}_\gamma^M \boldsymbol{q}_\gamma^M - \boldsymbol{w}_\gamma^M \boldsymbol{F} - \boldsymbol{c}^M \boldsymbol{w}_\gamma^M \boldsymbol{q}_\gamma^M = 0$$

$$\Rightarrow \pi_\gamma = (\frac{\sigma}{\sigma - 1} - 1) \boldsymbol{c}^M \boldsymbol{w}_\gamma^M \boldsymbol{q}_\gamma^M - \boldsymbol{w}_\gamma^M \boldsymbol{F} = 0$$

$$\Rightarrow \pi_\gamma = \frac{1}{\sigma - 1} \boldsymbol{c}^M \boldsymbol{w}_\gamma^M \boldsymbol{q}_\gamma^M - \boldsymbol{w}_\gamma^M \boldsymbol{F} = 0$$

$$\Rightarrow \pi_\gamma = \boldsymbol{w}_\gamma^M (\frac{\boldsymbol{c}^M \boldsymbol{q}_\gamma^M}{\sigma - 1} - \boldsymbol{F}) = 0 \qquad (2.68)$$

The equilibrium output for each firm

From (2.68), we can derive the equilibrium output for each firm (i.e., each variety) as

$$q^* = F(\sigma - 1)/c^M \qquad (2.69)$$

The equilibrium labor input

Substituting (2.69) into (2.62), we have the equilibrium labor input as

$$\begin{aligned} l^* &= F + c^M q^* \\ &= F + c^M [F(\sigma - 1)/c^M] \\ &= F + F(\sigma - 1) \\ &= F\sigma \end{aligned} \qquad (2.70)$$

➡ Both q^* and l^* are **constants** common to every active firm in all locations, that is, the optimal q^* and l^* are independent of the regions where the firms locate.

The equilibrium number of manufacturing firms

If L_γ^M is the number of manufacturing workers at location γ, and n_γ is the number of manufacturing firms (i.e., the number of the varieties produced) at γ , then

$$n_\gamma^* = L_\gamma^M / l^* = L_\gamma^M / F\sigma \qquad (2.71)$$

- The size of market (i.e., the size of consumption demand) affects neither the markup of price over marginal cost nor the scale at which individual goods are produced. [please refer both equations (2.67) and (2.69)]
- All scale effects work through changes in the variety of goods available.
- The Dixit-Stiglitz model implies that all market-size effect work through change in variety.

(The alternatives ways the economy takes advantage of the extent of the market is by producing at larger scale.)

⦿The Manufacturing Wage Equation

At each location γ , the **total sales** for every variety q_{γ}^{M} (2.61) is equivalent to the firm produce q^{*} (2.69), thus the following equation is satisfied:

$$q^{*} = \mu \sum_{s=1}^{R} Y_s (p_{\gamma}^{M})^{-\sigma} (T_{\gamma s}^{M})^{1-\sigma} G_s^{\sigma-1} \qquad (2.72)$$

$$(p_{\gamma}^{M})^{\sigma} = \frac{\mu}{q^{*}} \sum_{s=1}^{R} Y_s (T_{\gamma s}^{M})^{1-\sigma} G_s^{\sigma-1} \qquad (2.73)$$

Substituting (2.67) into (2.73), then we have

$$\left(\frac{\sigma}{\sigma-1}\cdot c^{M} w_{\gamma}^{M}\right)^{\sigma}=\frac{\mu}{q^{*}}\sum_{s=1}^{R} Y_{s}\left(T_{\gamma s}^{M}\right)^{1-\sigma} G_{s}^{\sigma-1}$$

$$w_{\gamma}^{M}=\left(\frac{\sigma-1}{\sigma c^{M}}\right)\left[\frac{\mu}{q^{*}}\sum_{s=1}^{R} Y_{s}\left(T_{\gamma s}^{M}\right)^{1-\sigma} G_{s}^{\sigma-1}\right]^{1/\sigma} \qquad (2.74)$$

From (2.74), we know that

The wage at location γ is higher, if

(1) the incomes in the firm's markets, Y_s , are higher,

(2) the firm's access to theses markets is better (lower $T_{\gamma s}^{M}$)

(3) the firm faces less competition in these markets,

from (2.59), we know $(n \downarrow \Rightarrow G_s \uparrow \Rightarrow w_{\gamma}^{M} \uparrow)$

⦿ The real wage

From "indirect utility function" (2.56), we (2.75)

$$V_\gamma = \mu^\mu (1-\mu)^{1-\mu} Y_\gamma G_\gamma^{-\mu} (p_\gamma^A)^{-(1-\mu)} \qquad (2.75)$$

Where Y_γ : denotes the nominal income (or wage) in location γ ,

Thus, the real wage of manufacturing workers in location, denoted γ , is given by ω_γ^M as follows:

$$\omega_\gamma^M = w_\gamma^M G_\gamma^{-\mu} (p_\gamma^A)^{-(1-\mu)} \qquad (2.76)$$

$G_\gamma^\mu (p_\gamma^A)^{1-\mu}$: define the cost-of –living index,

And if we normalize the price of agricultural good as equal one, $p_r^A = 1$,we can simplify the cost-of –living index as : G_γ^μ

In turn, the real wage and the indirect utility can be rewritten, respectively, as follows:

$$\omega_\gamma^M = w_\gamma^M G_\gamma^{-\mu} \qquad (2.77)$$

$$V_\gamma = \mu^\mu (1-\mu)^{1-\mu} w_\gamma G_\gamma^{-\mu} \qquad (2.78)$$

2.4.4 The Normalizations and Further Simplification

In order to simplify the previous equations, we can choose units such that the marginal labor requirement satisfies the following equation:

$$c^M = \frac{\sigma - 1}{\sigma} \qquad (2.79)$$

Substituting (2.78) into (2.69), (2.70), and (2.71), we have the results that

$$p_{\gamma}^{M} = w_{\gamma}^{M} \qquad (2.80)$$

$$q^{*} = l^{*} = \mu \qquad (2.81)$$

$$F = \mu/\sigma \qquad (2.82)$$

$$n_{\gamma} = L_{\gamma}^{M}/\mu \qquad (2.83)$$

Using equations (2.80)-(2.83), the manufacturing price index G_{γ} [shown as in equation (2.59)], and the nominal wage w_{γ}^{M} [shown as in equation (2.74)], are given, respectively, by

$$G_\gamma = [\sum_{s=1}^{R} n_s (p_s^M T_{s\gamma}^M)^{1-\sigma}]^{1/(1-\sigma)}$$

$$= [\frac{1}{\mu}\sum_{s=1}^{R} L_s^M (w_s^M T_{s\gamma}^M)^{1-\sigma}]^{1/(1-\sigma)} \qquad (2.84)$$

$$w_\gamma^M = (\frac{\sigma-1}{\sigma c^M})[\frac{\mu}{q^*}\sum_{s=1}^{R} Y_s (T_{s\gamma}^M)^{1-\sigma} G_s^{\sigma-1}]^{1/\sigma}$$

$$= [\sum_{s=1}^{R} Y_s (T_{s\gamma}^M)^{1-\sigma} G_s^{\sigma-1}]^{1/\sigma} \qquad (2.85)$$

➡We would use these two equations to examine both "the equilibrium of core and periphery" and "the equilibrium of symmetry", and also to investigate its stability on next chapter.