

國立嘉義大學101學年度基礎學科學力競賽試題卷:

科目: 經濟數學 題型:選擇題 配分:100%

1. List all the values of x for which the given function is not continuous.

$$f(x) = \begin{cases} x^3 & \text{if } x \leq 5 \\ 125 & \text{if } x > 5 \end{cases}$$

A) $x = 5$ B) $x = \pm 5$ C) The function is continuous for all values of x . D) $x = 0$.

2. Find $\lim_{x \rightarrow +\infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$. If the limiting value is infinite indicate whether it is $+$ or $-$.

$$f(x) = \frac{4 - 2x^3}{6x^3 + 6x + 1}$$

A) $+\infty, -\infty$ B) $-\infty, +\infty$ C) $-\frac{1}{3}, -\frac{1}{3}$ D) $-\frac{1}{3}, \frac{1}{3}$.

3. The cost of renting a backhoe at one distributor is \$325, plus \$35 per day. Write a linear function $C(x)$ that describes the cost of renting the backhoe for x days, then use your function to find how much it would cost to rent it for 12 days.

A) $C(x) = 35x + 313$; \$733 C) $C(x) = 12(325 + 35x)$; \$8,940
B) $C(x) = 325x + 35$; \$3,935 D) $C(x) = 325 + 35x$; \$745.

4. Find the equation of the tangent line to the graph of $f(x) = x^2 + 3$ at the point (3, 12).

A) $y = 12$ B) $y = 6x - 6$ C) $x = 3$ D) Not defined.

5. An object moves along a line in such a way that its position at time t is $s(t) = t^3 - 9t^2 + 15t + 2$. Find the velocity and acceleration of the object at time t . When is the object stationary?

A) $v(t) = 3t^2 - 18t + 15$; $a(t) = 6t - 18$; $t = 3$
B) $v(t) = 3t^2 - 6t + 15$; $a(t) = 6t - 6$; $t = 1$
C) $v(t) = 3t^2 - 18t + 15$; $a(t) = 6t - 18$; $t = 1$
D) $v(t) = 3t^2 - 18t + 15$; $a(t) = 6t - 18$; $t = 1$ and 5

6. Differentiate: $f(x) = (x^2 + 1)(x + 6)$

A) $x^2 + 1$ B) $3x^2 + 12x + 1$ C) $2x + 1$ D) $12x + 1$

7. An efficiency study of the morning shift at a certain factory indicates that an average worker arriving on the job at 7:00 A.M. will have assembled $f(x) = -x^3 + 7x^2 - 3x$ transistor radios x hours later. Approximately how many radios will the worker assemble between 9:00 and 9:30 A.M.?

A) Approximately 7 radios C) Approximately 14 radios
B) Approximately 13 radios D) Approximately 390 radios

8. Find $\frac{dy}{dx}$, where $xy^3 - 3x^2 = 7y$

A) $\frac{6x - y^3}{3xy^2 - 7}$ B) $y^3 - 6x$ C) $\frac{6x^2}{y^3}$ D) $y^3 - 6x - 7$

9. Determine where the graph of $f(x) = x^4 - 54x^2 + 6$ is concave up.

A) Everywhere B) For $x < -3$ and $x > 3$ C) Nowhere D) For $-3 < x < 3$

10. The demand function for a certain commodity is $D(p) = \frac{30}{p+5}$. For what values of p is the demand inelastic?

A) $p > -5$ B) $p > 0$ C) $p < -5$ D) $p < 0$

11. Use the second derivative test to find the relative maxima and minima of the function

$$f(x) = 2x^3 + 6x^2 - 18x + 1$$

- A) Relative maximum at $(-3, -8)$; relative minimum at $(1, 55)$
- B) Relative maximum at $(-3, 55)$; relative minimum at $(1, -9)$
- C) Relative maximum at $(-3, -9)$; relative minimum at $(-1, 23)$
- D) Relative maximum at $(1, 55)$; relative minimum at $(-3, -9)$

12. Locate all inflection points of $f(x) = \frac{x}{x^2 + 1}$.

- A) $(0, 0), \left(1, \frac{1}{2}\right), \left(-1, -\frac{1}{2}\right)$
- B) $(0, 0)$
- C) $(0, 0), \left(-1, -\frac{1}{2}\right)$
- D) $\left(-\sqrt{3}, -\frac{\sqrt{3}}{4}\right), (0, 0), \left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$

13. Find the intervals of increase and decrease for the function $f(x) = x^2 + 9x - 5$.

- A) Decreasing for $x < -\frac{9}{2}$ and increasing for $x > -\frac{9}{2}$
- B) Increasing for all x
- C) Decreasing for $x > -\frac{9}{2}$ and increasing for $x < -\frac{9}{2}$
- D) Decreasing for all x

14. If \$1,500 is invested at 9 percent compounded continuously, what is the balance after 13 years?

- A) \$4,598.71
- B) \$465.55
- C) \$1,635.00
- D) \$4,832.99

15. Suppose your family owns a rare book whose value t years from now will be $V(t) = 9e^{\sqrt{0.8t}}$ dollars. If

the prevailing interest rate remains constant at 6% per year compounded continuously, when will it be most advantageous for your family to sell the book and invest the proceeds? Round your answer to two decimal places.

- A) 125.00 years
- B) 48.61 years
- C) 194.44 years
- D) 55.56 years

16. Solve for x : $4 \ln x - \frac{1}{5} \ln x^4 = 16$

- A) $x = e$
- B) $x = e^{16}$
- C) $x = e^4$
- D) $x = e^5$

17. Differentiate the given function.

$$f(x) = 35 - 3e^{-0.06x}$$

- A) $3xe^{-0.06x}$
- B) $-0.18e^{-0.06x}$
- C) $0.18e^{-0.06x}$
- D) $-3xe^{-0.06x}$

18. Differentiate the given function.

$$f(x) = \ln x^3$$

- A) $\frac{x}{3}$
- B) $\frac{1}{3x}$
- C) $3x$
- D) $\frac{3}{x}$

19. Let $f(x) = 6x^5 - 90 \ln x$, for $x > 0$. Find the minimum value of f for $x > 0$.

- A) 0
- B) $3(3^5 - 15 \ln 3)$
- C) $18(1 - \ln 3)$
- D) $6(3^5 - 15 \ln 3)$

20. Evaluate $\int (5x^6 - 9x + 7) dx$

- A) $\frac{5x^7}{7} - \frac{9x^2}{2} + C$
- B) $\frac{5x^7}{7} - \frac{9x^2}{2} + 7x + C$
- C) $30x^5 - 9 + C$
- D) $5x^7 - 9x^2 + 7x + C$

21. A study indicates that x months from now the population of a certain city will be increasing at the rate of $(3 + 3x)x^{-1/2}$ people per month. By how much will the population increase over the next 9 months?

- A) 54 people
- B) 62 people
- C) 72 people
- D) 82 people

22. Evaluate $\int 5x^4 e^{x^5} dx$

- A) $e^{x^6} + C$
- B) $x^5 e^{x^5} + C$
- C) $25x^5 e^{x^5} + C$
- D) $e^{x^5} + C$

23. Evaluate $\int x^6 \sqrt{x^7 + 4} dx$

- A) $x^7 + 4 + C$ B) $C(x^7 + 4)^{3/2}$ C) $\frac{2(x^7 + 4)^{3/2}}{21} + C$ D) $(x^7 + 4)^{3/2} + C$

24. Solve the given initial value problem for $y = f(x)$.

$$\frac{dy}{dx} = \frac{x+5}{x^2+10x+3} \quad \text{where } y = 6 \text{ when } x = 1$$

- A) $y = \frac{1}{2} \ln|x^2 + 10x + 3| - \frac{1}{2} \ln 14 + 6$ C) $y = \frac{1}{2} \ln|x^2 + 10x + 3| + 6$
B) $y = \frac{1}{2} \ln|x^2 + 10x + 3| - \frac{1}{2} \ln 14$ D) $y = \frac{1}{2} \ln|x^2 + 10x + 3|$

25. A company makes a certain product for \$4 each and sells it for \$8. If the company has overhead expenses of \$10,000 per year, how many of its products must be made and sold to break even?

- A) 40,000 B) 2,500 C) 20,000 D) 10,000