

國立嘉義大學 101 學年度基礎學科學力競賽試題卷

科目：微積分

題型：選擇題

配分：100%

1. Evaluate the limit, if it exists

$$\lim_{x \rightarrow -2} \frac{x+2}{x^3+8} = \text{_____}.$$

- (A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{1}{12}$

(D) Does not exist

2. Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 < x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$$

- (A) $a = \frac{1}{2}, b = \frac{1}{2}$ (B) $a = \frac{5}{2}, b = \frac{1}{2}$ (C) $a = \frac{-1}{2}, b = \frac{-1}{2}$
 (D) $a = \frac{1}{2}, b = \frac{5}{2}$

3. Let

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x \leq 2 \\ x - 1 & \text{if } x > 2 \end{cases}, \text{ Find } \lim_{x \rightarrow 2} f(x) = ?$$

- (A) 0 (B) 1 (C) $\frac{-1 \pm \sqrt{21}}{2}$

(D) Does not exist

4. For what values of c does the curve have maximum and minimum points?

$$F(x) = 5x^3 + cx^2 + 10x$$

Select the correct answer.

- (A) $|c| > 15;$ (B) $|c| > \sqrt{150};$ (C) $|c| > 1500;$
 (D) $|c| > \sqrt{30};$

5. If $y = 2x^3 + 6x$ and $\frac{dx}{dt} = 6$, find $\frac{dy}{dt}$ when $x = 5$.

Select the correct answer.

- (A) 936 (B) 946 (C) 956
 (D) None of these

6. Find an equation of the tangent to the circle $x^2 + y^2 = 25$ at the point P(3, 4).

- (A) $4x - 3y = 0$ (B) $3x - 4y + 7 = 0$ (C) $4x + 3y - 24 = 0$
 (D) $3x + 4y - 25 = 0$

7. Find the absolute maximum (M) and minimum (m) values of the function on the given interval.

$$f(x) = x^3 - 3x^2 + 1, -\frac{1}{2} \leq x \leq 4. \text{ Select the correct answer.}$$

- (A) M>0, m>0. (B) M>0, m<0. (C) M<0, m<0
 (D) M + m = 0

8. Find the inflection points for the function given.

$$f(x) = 8x + 2 - \sin x, 0 < x < 3\pi$$

Select the correct answer.

- (A) $(\pi, 8\pi), (2\pi, 16\pi + 2);$ (B) $(\pi, 2), (2\pi, 16\pi + 2);$ (C) $(\pi, 8\pi), (2\pi, 16\pi);$
 (D) $(\pi, 8\pi + 2), (2\pi, 16\pi + 2);$

9. Find the area of the region enclosed by the line $y = 4x$ and the curve $y = x^3 + 3x^2$.

- (A) 32.75 (B) 31.25 (C) 30.75
 (D) None of these.

10. Find the volume of the solid of revolution formed by rotating the region R about the x axis, R is the region under the curve $y = \sqrt{x}$ and above the X-axis from $x = 1$ to $x = 4$.

- (A) $\frac{11\pi}{2}$ (B) $\frac{13\pi}{2}$ (C) $\frac{15\pi}{2}$
 (D) $\frac{17\pi}{2}$ 《背面尚有試題》

11. Let $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = K$. Select the correct answer.

- (A) $K=1$
(B) $0 < K < 1$
(C) $1 < K < 2$
(D) $2 < K < 3$

12. Evaluate the definite integral $\int_0^1 \left(\frac{6t}{t^2 + 1} \right) dt$ using the fundamental theorem of calculus.

- (A) $6\ln 2$
(B) $3\ln 2$
(C) $6 + 2\ln 2$
(D) $3 + \ln 2$

13. $\int_0^\infty \frac{1}{1+x^2} dx = ?$

- (A) $\frac{\pi}{2}$
(B) $\frac{\pi}{4}$
(C) π
(D) divergent

14. $\int_{-1}^1 \frac{1}{x} dx = ?$

- (A) $\ln 2$
(B) 0
(C) $-\ln 2$
(D) divergent

15. $\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^2} dx = ?$

- (A) $\frac{\pi}{2}$
(B) $\frac{\pi}{4}$
(C) π
(D) ∞

16. Evaluate the definite integral $\int_{-1}^1 \left(\frac{1}{e^x} - \frac{1}{e^{-x}} \right) dx$ using the fundamental theorem of calculus.

- (A) $2e$
(B) $\frac{2}{e}$
(C) 0
(D) $2\left(e - \frac{1}{e}\right)$

17.

For what values of p is the integral $\int_1^\infty \frac{1}{x^p} dx$ convergent?

- (A) $P < 1$
(B) $P = 1$
(C) $P > 1$
(D) $P > 0$.

18. Find $\int e^{\sin x} \cos x dx = ?$

- (A) $e^{\cos x} + c$
(B) $e^{\sin x} + c$
(C) $\sin x e^{\cos x} + c$
(D) $\cos x e^{\sin x} + c$

19. $\int_0^5 2^x dx = ?$

- (A) $31(\ln 2)$
(B) $\frac{31}{\ln 2}$
(C) $31 + \ln 2$

- (D) 31

20. $\int \frac{x^3 + x}{x-1} dx = ?$

- (A) $\frac{x^3}{3} + \frac{x^2}{3} + 2x + 2\ln|x-1| + c$
(B) $\frac{-x^3}{2} + \frac{x^2}{3} - 2x + 2\ln|x-1| + c$
(C) $\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\ln|x-1| + c$
(D) $\frac{-x^3}{2} + \frac{x^2}{3} + 2x + 2\ln|x+1| + c$

《試題結束請將答案卡及試題卷一併繳回》