國立嘉義大學九十二學年度轉學生招生考試試題

科目：線性代數

一、填充題： (60%，請標明題號，並將答案寫在答案卷上)

1. If $(1, 2, -3)$, $(2, -1, a)$, $(0, 2a+1, -8)$ are linearly dependent, then the integer value of a is __________ . (10%)

2. Let $A=\begin{bmatrix} 3 & -2 & -5 \\ 4 & -1 & -5 \\ -2 & -1 & -3 \end{bmatrix}$.

(1). The characteristic polynomial of A is _______ . (3%)

(2). The eigenvalues of A are _______ . (2%)

(3). The eigenvectors of A are _______ . (4%)

(4). Is A diagonalizable? _______ . (1%)

3. Let $A=\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$.

(1). The minimal polynomial of A is _______ . (5%)

(2). $A^5-4A^4+7A^3-9A^2+6A-I_3=$ _______ . (5%)

4. (1). Let $B=\begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}$ and $f(x)=2x^2-4x+3$. The linearly independent eigenvectors of B are _______ , (4%) $f(B)=$ _______ . (3%)

(2). If $\mathbf{x}$ is an eigenvector of A^2, then determine whether $\mathbf{x}$ is also an eigenvector of A? _______ . (3%).

5. Let $A=\begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & \frac{1}{4} \\ 0 & 1 \end{bmatrix}$.

(1). $\text{rank}(A)=$ ________ . (2%), nullity of A = _________ . (2%)

(2). Is B one-to-one? _______ . (3%) Is B onto? _______ . (3%)

6. (1). Let $A^2=O$. All the eigenvalues of A are ________. (3%)

(2). Let A be an orthogonal matrix. Determine $|A|=$ ________ . (3%)

(3). If 0 is an eigenvalue of A, determine whether A is singular or nonsingular? ________ . (4%)

二、計算證明題： (40%，請標明題號，並將計算過程寫在答案卷上)

1. Let A be a real $n\times n$ matrix. Show that if 1 is an eigenvalue of A, then A^k-I_n is not invertible, where I_n is the identity matrix and k is a positive integer. (10%)

2. Consider the bases $A=\{(2, 4), (3, 1)\}$ and $B=\{(1, 1), (1, -1)\}$. Suppose that $T:\Re^2\rightarrow\Re^2$ is a linear transformation such that the matrix of T with respect to A is $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, find the matrix of T with respect to B. (10%)

3. Let $S=\{\mathbf{u}, \mathbf{v}\}$ be a linearly independent set. Prove that the set $\{\mathbf{u}+\mathbf{v}, \mathbf{u}-\mathbf{v}\}$ is linearly independent. (10%)

4. Let V be the vector space of functions which has $\{\sin\theta, \cos\theta\}$ as a basis, and let D be the differential operator on V.

(1). Determine the matrix A of D. (5%)

(2). Show that D is a zero of $f(t)=t^2+1$. (5%)