國立嘉義大學九十三學年度轉學生招生考試試題

科目：線性代數

一、填充題：60%　(請標明題號，並將答案寫在答案卷上)

1. Let the matrices X, Y, Z and W be given by

$$X=\begin{bmatrix}1\\2\\0\\1\end{bmatrix},\ Y=\begin{bmatrix}-1\\0\\3\\2\end{bmatrix},\ Z=\begin{bmatrix}3\\4\\-1\\2\end{bmatrix},\ W=\begin{bmatrix}3\\2\\-4\\-1\end{bmatrix}.$$

(a).Find scalars a, b, and c such that $W = aX + bY + cZ$.

Ans.：a=______.b=______.c=_____.（6%）

(b).Find scalars a and b such that $Z = aX + bY$.　Ans.：a=______.b=______.（4%）

2. (a). Let A be an $n \times n$ idempotent matrix, i.e. $A^2 = A$. Find all the eigenvalues of A.

Ans.：_______.　(4%)

(b). Determine all $n \times n$ symmetric matrices that have 0 as their only eigenvalue.

Ans.：_______.　(3%)

(c). Let 0 be an eigenvalue of A. Is A singular or nonsingular? Ans.：______.（3%）

3. For what values of a does the matrix $A=\begin{bmatrix}0 & 1\\ a & 1\end{bmatrix}$ have the following characteristics?

(a). A has an eigenvalue of multiplicity 2. Ans.：a=______.（4%）

(b). A has -1 and 2 as eigenvalues.　Ans.：a=______.（3%）

(c). A has real eigenvalues.　Ans.：a=______.（3%）

4. Let $A=\begin{bmatrix}3 & 1 & -2\\ -1 & 0 & 5\\ -1 & -1 & 4\end{bmatrix}$.

(a). The determinant of A is __________.　(3%)

(b). The characteristic polynomial of A is __________.　(3%)

(c). The eigenvalues of A are __________.　(4%)

5. Suppose that $T:\Re^2 \to \Re^2$ is linear, $T(1,\ 0)=(1,\ 4)$, and $T(1,\ 1)=(2,\ 5)$.

(a). What is $T(2,\ 3)$? Ans.：_____ .(6%)

(b). Is T one-to-one? Ans.：_____. (2%)

(c). Is T onto? Ans.：_____. (2%)

6. Let $A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(a). The range of A is _____. (3%)

(b). $Rank(A)=$ ________ . (2%)

(c). The null space of A is ________ . (3%)

(d). Nullity of $A=$ ________ . (2%)

二、計算證明題：40% (請標明題號，並將計算過程寫在答案卷上)

1. Show that $\begin{vmatrix} a & 1 & 1 & 1 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & 1 \\ 1 & 1 & 1 & a \end{vmatrix} = (a+3)(a-1)^3$. （10%）

2. Let $T:V \to U$ and $S:U \to W$ be linear transformations.

(a). Prove that if S and T are both one-to-one than so does $S \circ T$. （3%）

(b). Prove that the kernel of T is contained in the kernel of $S \circ T$. （3%）

(c). Prove that if $S \circ T$ is onto, then so is S. （4%）

3. Suppose that A is similar to B. Prove that the eigenvalues of A are the same as the eigenvalues of B. (10%)

4. Let $S=\{u,\ v\}$ be a linearly independent set. Prove that the set $\{u+2v,\ 2u-v\}$ is linearly independent. (10%)