7   70

1. $\displaystyle\lim_{n\to\infty}\left(\frac{n+1}{n+2}\right)^n =$ _____ .

2. Assume that f is a continuous function and that $\displaystyle\int_0^x tf(t)\,dt = \sin x - x\cos x$.

    (a) $f(\pi/2) =$ _____ ,   (b) $f'(x) =$ _____ .

3. $\displaystyle\int \frac{\sec^2\theta}{\tan^3\theta - \tan^2\theta}\,d\theta =$ _____ .

4. The arc length from the origin to the point $(x(\theta_1),\,y(\theta_1))$ along the exponential spiral

    $r = a e^{c\theta}$, $a, c > 0$, is _____ .

5. Consider the region in the right half-plane that is outside the parabola $y = x^2$ and is

    between the lines $y = x+2$ and $y = 2x-2$. A solid is generated by revolving the specified

    region around the y-axis. The volume of the solid is _____ .

6. $\displaystyle\lim_{x\to 1}\frac{1}{x-1}\int_{x+1}^{x^2+x}\ln(t^2+1)\,dt =$ _____ .

7. Let $f(x) = \begin{cases} x\,e^{-\frac{1}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Then $f''(0) =$ _____

8. $\displaystyle\int x^2\, 2^{x^3}\,dx =$ _____ .

9. Let V be the volume of the solid generalized by revolving the region between $y = \sqrt{x}$

    and $y = x^2$, $0 \le x \le 1$ about the line $x = 2$. Then $V =$ _____ .

10. Let $\dfrac{dy}{dx} = \dfrac{2x}{y}$, $y(0) = 2$. Then $y =$ _____ .

10   30

1. Let $F(x) = \displaystyle\int_0^x t(t-3)^2\,dt$.

    (a) Find the critical numbers of F and determine the intervals on which F is

        increasing and the intervals on which F is decreasing.

    (b) Determine the concavity of the graph of F and find the points of inflection, if any.

    (c) Sketch the graph of F.

2. Evaluate $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-(x-y)^2}}{1+(x+y)^2}\,dx\,dy$ by intergrating over the square $S_a: -a \le x \le a,\ -a \le y \le a$

    and taking the limit as $a \to \infty$.

3. Find the minimum values of $f(x,\,y,\,z) = 2x^2 + y^2 + 3z^2$ subject to the constraint

    $2x - 3y - 4z - 49 = 0$.