國立嘉義大學九十六學年度

應用經濟學系碩士班招生考試試題

科目：統計學

計算題及證明題：（每題 25 分，共 100 分）

1. Let X be a random variable with mean μ and variance σ^2. Three independent observations are drawn from this distribution: χ_1, χ_2, and χ_3. Consider the three different estimators of μ, $\hat{\mu}_1 = 0.2\chi_1 + 0.3\chi_2 + 0.5\chi_3$, $\hat{\mu}_2 = 0.4\chi_1 + 0.2\chi_2 + 0.4\chi_3$, and $\hat{\mu}_3 = 0.3\chi_1 + 0.3\chi_2 + 0.3\chi_3$.

   (a) Prove that the first two are unbiased estimators but the last one is biased. (5 分)

   (b) Express the variances of the first two estimators (ignore the third) in terms of σ^2. (10 分)

   (c) Which one is more efficient and why? (10 分)

2. In the model $Y_t = \alpha + \beta X_t + u_t$ an estimate of β is obtained as follows:

$$\hat{\beta} = \frac{1}{n-1}\sum_{t=2}^{t=n}\left[\frac{Y_t - Y_{t-1}}{X_t - X_{t-1}}\right]$$

   (a) Give a geometric interpretation of $\hat{\beta}$. (5 分)

   (b) Show that $\hat{\beta}$ is unbiased and consistent. Be sure to state the assumptions needed to prove this. (10 分)

   (c) Without actually deriving the variance of $\hat{\beta}$, argue why this estimator is inefficient relative to the OLS estimator of β. (10 分)

3. The production manager at Bellevue Steel, a manufacturer of wheelchairs, wants to compare the number of defective wheelchairs produced on the day shift with the number on the afternoon shift. A sample of the production from 6 days shifts and 8 afternoon shifts revealed the following number of defects.

Day	5	8	7	6	9	7		
Afternoon	8	10	7	11	9	12	14	9

At the 0.05 significance level, is there a difference in the mean number of defects per shift?

   (a) State the null hypothesis and the alternate hypothesis. (4 分)

   (b) What is the decision rule? (4 分)

   (c) What is the value of the test statistic? (4 分)

   (d) What is your decision regarding the null hypothesis? (4 分)

   (e) Interpret the result. (4 分)

   (f) What are the assumptions necessary for this test? (5 分)

4. Rudduck Shampoo sells three shampoos, one each for dry, normal, and oily hair.

   Sales, in millions of dollars, for the past five months are given in the following table. Using the 0.05 significance level, test whether the mean sales differ for the three types of shampoo or by month. (25 分)

Sales($million)			
Month	**Dry**	**Normal**	**Oily**
June	7	9	12
July	11	12	14
August	13	11	8
September	8	9	7
October	9	10	13