國立嘉義大學九十五學年度

管理研究所碩士班招生考試（乙組）試題

科目：應用微積分

一、 Suppose that the supply function for some commodity is $S(q)=q^2+5q+100$ and the demand function for the commodity is $D(q)=350-q^2$. (15%)
Find：(a) the producers' surplus and (b) the consumers' surplus.

二、Let $P(5,-12)$ be a point on the circle $x^2+y^2=169$.(20%)
(a) What is the slope of the line joining P and O(0,0)？
(b) Find an equation of the tangent line to the circle at P.
(c) Let $Q(x,y)$ be another point on the circle in the fourth quadrant. Find the slope m_x of the line joining P and Q in terms of x.
(d) Calculate $\lim_{x\to 5^+} m_x$. How does this number relate to your answer in part (b)？

三、A baseball diamond is a square, 90 feet on a side. A runner runs from second base to third at 20 ft/sec . How fast is the distance S between the runner and home base changing when he is 15 feet from third base？(15%)

四、某公司估計每週製造某產品 x 單位的成本(元)為 $C(x)=x^3-3x^2-80x+500$，每一單位的售價為 2,800 元。問每週生產若干單位可使利潤最大？又每週之最大可能利潤為何？(10%)

五、假定某甲對貨品 X 與 Y 之效用函數(Total Utility Function)為
TU=$16X-X^2+5Y-2Y^2+XY^2$ ，(10%)
(a) 試導出某甲對貨品 X 與 Y 的邊際效用(Marginal Utility)函數。
(b) 當 X=5 單位，Y=2 單位時， MU_x 為多少？又當 X=2 單位，Y=5 單位時，MU_y 為多少？

六、試計算下列各題：(30%)

(a) $\int x^2e^x dx$

(b) $\int_{-\infty}^{0} \frac{1}{(1-2x)^{3/2}}dx$

(c) $\int \frac{x^5+x-1}{x^4-x^3}dx$

(d) $\int_0^1\int_0^{1-x} (x^2+\frac{1}{3}xy)dydx$

(e) $\int_0^3 |2x-3|dx$

(f) $\lim_{x\to\infty} \frac{\log_2 x}{\log_3(x+3)}$