國立嘉義大學九十五學年度
數學教育研究所碩士班招生考試試題

科目：微積分

1. Let the function $f(x) = \begin{cases} x^2 + x - 1 & \text{if } x \text{ is irrational} \\ 3x - 2 & \text{if } x \text{ is rational} \end{cases}$.

   (a) Show that f is continuous at 1. （10%）

   (b) Is f differentiable at 1？Justify your answer.（10%）

2. Find the volume of the solid formed by revolving the region bounded by $f(x) = 2 - x^2$ and $g(x) = 1$ about the line $y = 1$.（20%）

3. Let $y = \ln(\ln(\ln x))$. Find $\dfrac{dy}{dx}$.（10%）

4. Find the power series centered at 0 for $\dfrac{1}{\sqrt{4+x^2}}$ and the radius of convergence of the series.（20%）

5. Let R be the annular region lying between the two circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 5$. Evaluate the integral $\iint\limits_R (x^2 + y)\, dA.$（20%）

6. Given $5x^2z - 3x^2y^2 + 2y^2z^5 + 2z = 5,$ find $\dfrac{\partial z}{\partial x}$.（10%）