國立嘉義大學九十六學年度
數學教育研究所碩士班招生考試試題

科目：微積分

1. Let $f(x)=\begin{cases} x+\sqrt[3]{x}, & \text{if } x\geq 8 \\ mx+b, & \text{if } x<8. \end{cases}$ Determine the constants m and b so that the function f is differentiable everywhere. (10%)

2. Determine the constants a and b so that $\lim_{x\to 1}\frac{\sqrt[3]{ax+2b}+2}{x-1}=2.$ (10%)

3. Find (a) $\lim_{x\to\infty} x(e^{\frac{1}{x}}-1).$ (10%) (b) $\lim_{x\to 0}\int_0^{2x}\frac{\sin(3t)}{5t}\,dt.$ (10%)

4. Find $\lim_{n\to\infty}\dfrac{\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\Lambda+\frac{1}{\sqrt{n}}}{\frac{n}{1^2}+\frac{n}{2^2}+\Lambda+\frac{n}{n^2}}.$ (10%)

5. (a) Analyze and sketch the graph of $y=\frac{x^3}{6}+\frac{1}{2x},\ x>0.$ (10%)

   (b)Find the arc length of the graph of $y=\frac{x^3}{6}+\frac{1}{2x}$ on the interval $[1,2]$. (10%)

6. (a) Let $y=2^{3x}$. Find $\frac{dy}{dx}$. (10%)

   (b) Let $\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{\frac{-t^2}{2}}\,dt=1$. Find the integral: $\int_0^{\infty}\frac{1}{\sqrt{2x}}\,e^{-x}\,dx.$ (10%)

7. Evaluate $\lim_{n\to\infty}\left(\frac{n-2}{n+2}\right)^n$. (10%)