

**國立嘉義大學九十三年學年度**  
**數學教育研究所碩士班招生考試試題**  
**科目：微積分**

1. Find the volume of the solid generalized by revolving the region between

$$y = \sqrt{x} \text{ and } y = x^2, \quad 0 \leq x \leq 1 \text{ about the line } x = 3. \quad (20 \text{ 分})$$

2. Evaluate  $\lim_{x \rightarrow 1} \frac{1}{x-1} \int_{x+1}^{x^2+x} \ln(t^2 + 3) dt$ . (20 分)

3. Let  $\frac{dy}{dx} = \frac{4x}{y}$ ,  $y(0) = 2$ . Find the equation of  $y$ . (10 分)

4. Solve  $e^{2x} = 2e^x + 1$ . (10 分)

5. Suppose that the Celsius temperature at the point  $(x, y, z)$  on the sphere

$x^2 + y^2 + z^2 = 1$  is  $T = 300x^2yz$ . Find the highest temperature on the sphere. (20 分)

6. Suppose that a function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is continuous at a real number  $x_0$

and  $f(x_0) > 0$ . Show that there exists a positive number  $\delta$  such that

$f(x) > 0$ , for all  $x \in (x_0 - \delta, x_0 + \delta)$ . (20 分)